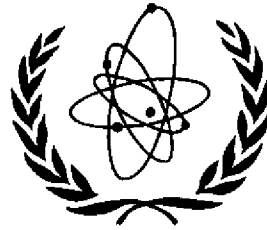


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PHYSICS

ON THE ALGEBRA OF  $SU(6)$

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# ON THE ALGEBRA OF $SU_6$

by

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The remarkable success of  $SU_6$  ideas [1] in elementary particle physics makes it imperative to look for its relativistic basis. Consider the free Dirac Lagrangian  $\mathcal{L} = \bar{\psi} (\not{p} - m) \psi$  for a single particle.  $\mathcal{L}$  is invariant for the Pauli-Lubanski transformation

$$\psi' = (1 + i E_{\mu} \omega_{\mu}) \psi \quad (1)$$

where

$$\omega_{\mu} = \frac{1}{4} E_{\mu\nu\rho\kappa} \sigma_{\nu\rho} p_{\kappa}$$

Since  $p_{\mu} \omega_{\mu} \equiv 0$ , there are three independent generators with the [2] commutation relation

$$[\omega_{\mu}, \omega_{\nu}] = i E_{\mu\nu\rho\kappa} p_{\rho} \omega_{\kappa} \quad (2)$$

The generators give rise to an  $SU_2$ -like (in general non-compact) structure which satisfies for the spin 1/2 case the anti-commutation relation:

$$\begin{aligned} \{\omega_{\mu}, \omega_{\nu}\} &= -\frac{1}{4} (\gamma_5 [\delta_{\mu\nu}, \not{p}], \gamma_5 [\not{p}, \delta_{\mu\nu}]) \\ &= 2 (p_{\mu} p_{\nu} - p^2 g_{\mu\nu}) \end{aligned} \quad (3)$$

Consider now the case when  $\psi$  is a three-component Sakata-like entity (representing quarks). It is possible to extend (1) to the general ( $SU_6$ ) transformation:

$$\psi' = (1 + i E^i T^i + i E_{\mu}^{\alpha} T^{\alpha} \omega_{\mu}) \psi \quad (4)$$

Here  $T^{\alpha} \begin{pmatrix} \alpha=0, \dots, 8 \\ i=1, \dots, 8 \end{pmatrix}$  are the usual  $U_3$  generators with  $T^0 = 1$  and from (2),

$$\begin{aligned}
[T^a \omega_\mu, T^b \omega_\nu] &= \frac{1}{2} \{ \omega_\mu, \omega_\nu \} [T^a, T^b] + \frac{1}{2} [\omega_\mu, \omega_\nu] \{ T^a, T^b \} \\
&= i (\rho_{\mu\nu} - \rho^2 g_{\mu\nu}) c_{ijk} T^k + \frac{i}{2} \epsilon_{\mu\nu\rho\kappa} \rho_\rho \omega_\kappa \left( \frac{1}{3} \delta_{ij} T^0 + d_{ijk} T^k \right) \quad (5) \\
[T^i \omega_\mu, T^j] &= \frac{i}{2} \omega_\mu c_{ijk} T^k
\end{aligned}$$

The adjoint representation-densities are given by  $\bar{\psi} \gamma_\mu \omega_\nu T^a \psi$  and  $\bar{\psi} \gamma_\mu T^i \psi$  which satisfy as usual,

$$\bar{\psi} \not{\omega}_\nu T^a \psi = \bar{\psi} \not{T}^i \psi = 0 \quad (6)$$

One may now generalize the case of  $SU_6$  above to the more general case [3]  $(SU_6)_L \times (SU_6)_R$ ; i.e., start with the fields  $\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$ . Clearly  $m\bar{\psi}\psi$  term is not invariant for the full group (though the invariance is unaffected for the pure  $\omega_\mu$  transformations). There are altogether now 70 generators  $\bar{\psi}_{L,R} \gamma_\mu \omega_\nu T^a \psi_{L,R}$ ,  $\bar{\psi}_{L,R} \gamma_\mu T^i \psi_{L,R}$ . The conservation equations (6) however need modifying; thus:

$$\begin{aligned}
\bar{\psi} \not{\omega}_\nu \gamma_5 T^a \psi &\neq 0 = (2m \bar{\psi} \omega_\nu \gamma_5 T^a \psi) \\
\bar{\psi} \not{\gamma}_5 T^i \psi &\neq 0 = (2m \bar{\psi} \gamma_5 T^i \psi).
\end{aligned}$$

From this point of view the  $0^-, 1^-$  35-fold (represented by the field operators  $\bar{\psi} \omega_\nu \gamma_5 T^a \psi$  and  $\bar{\psi} \gamma_5 T^i \psi$ ) is a remnant of the broken  $(SU_6)_L \times (SU_6)_R$  symmetry.

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References

[1] F. Gürsey and L.A. Radicati, Phys. Letters, 13, 5, 173 (1964);  
A. Pais, *ibid*, 13, 5, 175 (1964); B. Sakita, Phys. Letters, to  
be published.

[2] By the usual procedure one constructs the conserved current-  
density  $\bar{\psi} \gamma_{\mu} \omega_{\nu} \psi$  so that a representation for  $\omega_{\nu}$  is given  
by

$$\int d^3x \bar{\psi} \gamma_4 \omega_{\nu} \psi.$$

In checking the C.R. (2), (5) and (6) care is needed in writing  
anti-commutators like  $\{\bar{\psi}(x), \psi(y)\} \delta(x_0 - y_0)$ .

[3] This is analogous to study of the extended Algebras  $(SU_3)_L \times$   
 $(SU_3)_R$  by A. Salam and J. C. Ward (Il Nuovo Cimento, 19, 167  
(1961)), M. Gell-Mann (Phys. Rev. 125, 1067 (1962)) and  
Y. Nambu and P. Freund (Phys. Rev. Letters, 12, 714 (1964)).