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ON THE ALGEBRA OF SU(6)

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by

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The remarkable success of SU_6 ideas [1] in elementary particle physics makes it imperative to look for its relativistic basis. Consider the free Dirac Lagrangian $\mathcal{L} = \bar{\psi}(\not{p} - m)\psi$ for a single particle. $\mathcal{L}$ is invariant for the Pauli-Lubanski transformation

$$\psi' = (1 + i E_\mu \omega_\mu)\psi \tag{1}$$

where

$$\omega_\mu = \frac{1}{4} \epsilon_{\mu\nu\rho\kappa} \sigma_{\nu\rho} p_\kappa$$

Since $p_\mu \omega_\mu \equiv 0$, there are three independent generators with the [2] commutation relation

$$[\omega_\mu, \omega_\nu] = i\, \epsilon_{\mu\nu\rho\kappa}\, p_\rho\, \omega_\kappa \tag{2}$$

The generators give rise to an SU_2-like (in general non-compact) structure which satisfies for the spin 1/2 case the anti-commutation relation:

$$\begin{aligned}\{\omega_\mu, \omega_\nu\} &= -\frac{1}{4}\left(\gamma_5[\gamma_\mu, \not{p}],\ \gamma_5[\gamma_\nu, \not{p}]\right) \\ &= 2\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\end{aligned} \tag{3}$$

Consider now the case when ψ is a three-component Sakata-like entity (representing quarks). It is possible to extend (1) to the general (SU_6) transformation:

$$\psi' = (1 + i E^i T^i + i E_\mu^\alpha T^\alpha \omega_\mu)\psi \tag{4}$$

Here $T^\alpha \begin{pmatrix}\alpha = 0, \cdots, 8 \\ i = 1, \cdots, 8\end{pmatrix}$ are the usual U_3 generators with $T^0 = 1$ and from (2),

$$[T^\alpha \omega_\mu, T^\beta \omega_\nu] = \frac{1}{2}\{\omega_\mu, \omega_\nu\}[T^\alpha, T^\beta] + \frac{1}{2}[\omega_\mu, \omega_\nu]\{T^\alpha, T^\beta\}$$

$$= i(p_\mu p_\nu - p^2 g_{\mu\nu}) c_{ijk} T^k + \frac{i}{2} \epsilon_{\mu\nu\rho\kappa} p_\rho \omega_\kappa \left(\frac{1}{3}\delta_{ij} T^0 + d_{ijk} T^k\right) \qquad (5)$$

$$[T^i W_\mu, T^j] = \frac{i}{2} \omega_\mu c_{ijk} T^k$$

The adjoint representation-densities are given by $\bar{\psi} \gamma_\mu \omega_\nu T^\alpha \psi$ and $\bar{\psi} \gamma_\mu T^i \psi$ which satisfy as usual,

$$\bar{\psi} \not{p} \omega_\nu T^\alpha \psi = \bar{\psi} \not{p} T^i \psi = 0 \qquad (6)$$

One may now generalize the case of SU_6 above to the more general case [3] $(SU_6)_L \times (SU_6)_R$; i.e., start with the fields $\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$ Clearly $m\bar{\psi}\psi$ term is not invariant for the full group (though the invariance is unaffected for the pure ω_μ transformations). There are altogether now 70 generators $\bar{\psi}_{L,R} \gamma_\mu \omega_\nu T^\alpha \psi_{L,R}$, $\bar{\psi}_{L,R} \gamma_\mu T^i \psi_{L,R}$. The conservation equations (6) however need modifying; thus:

$$\bar{\psi} \not{p} \omega_\nu \gamma_5 T^\alpha \psi \neq 0 = (2m \, \bar{\psi} \omega_\nu \gamma_5 T^\alpha \psi)$$

$$\bar{\psi} \not{p} \gamma_5 T^i \psi \neq 0 = (2m \, \bar{\psi} \gamma_5 T^i \psi).$$

From this point of view the 0^-, 1^- 35-fold (represented by the field operators $\bar{\psi} \omega_\nu \gamma_5 T^\alpha \psi$ and $\bar{\psi} \gamma_5 T^i \psi$) is a remnant of the broken $(SU_6)_L \times (SU_6)_R$ symmetry.

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References

[1] F. Gürsey and L.A. Radicati, Phys. Letters, 13, 5, 173 (1964); A. Pais, ibid, 13, 5, 175 (1964); B. Sakita, Phys. Letters, to be published.

[2] By the usual procedure one constructs the conserved current-density $\bar{\psi}\gamma_\mu \omega_r \psi$ so that a representation for ω_r is given by
$\int d^3x \;\bar{\psi}\gamma_4 \omega_r \psi$.
In checking the C.R. (2), (5) and (6) care is needed in writing anti-commutators like $\{\bar{\psi}(x), \not{p}\,\psi(y)\}\,\delta(x_0-y_0)$.

[3] This is analogous to study of the extended Algebras $(SU_3)_L \times (SU_3)_R$ by A. Salam and J. C. Ward (Il Nuovo Cimento, 19, 167 (1961)), M. Gell-Mann (Phys. Rev. 125, 1067 (1962)) and Y. Nambu and P. Freund (Phys. Rev. Letters, 12, 714 (1964)).