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ABDUS SALAM

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PIAZZA OBERDAN

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SYMMETRIES OF STRONG INTERACTIONS

My first task is to thank all the authors who have contributed to this session. The first slide (Fig.1) shows their names. The Russian language has a good phrase for such a slide: "bratskaya mogila" - "the friendly communal grave". There is not a single important theoretical idea I shall report on today which has not been expressed by at least two groups of authors. It is not commonly recognized, but in a real sense Theoretical Physics has become as much a group-endeavour as Experimental Physics. If, in mentioning names, I happen to omit some by inadvertence, I beg for your indulgence.

After years of frustration and failure, it is always fun to report on a story of comparative success. For even the most sceptical of us cannot deny that the use of group-theoretic ideas has paid a handsome dividend to the symmetry physicist. My report shall naturally therefore have a strong group-theoretic bias.

I shall discuss:

<u>First</u> The successful tests of SU_3 (To its failures I shall turn a blind eye).

<u>Second</u> The composite models of elementary particles based on triplet models.

<u>Third</u> Group extensions and super-symmetries like $SU_3 \times SU_3$.

<u>Fourth</u> Dynamical considerations.

I. Tests of the Unitary Symmetry

The eight-fold way [1] has to its credit a small but impressive number of successful tests. These are:

(A) The existence of nearly pure multiplets containing 1, 8 and 10 particles of the same spin and parity. The positively identified nearly pure multiplets are the 0^- and $\frac{1}{2}^+$ octets and one $3/2^+$ decuplet.

(B) <u>The Mass Formulae</u>

Assuming that SU_3 symmetry is broken and symmetry-breaking can be treated as a small perturbation, one gets the well-known set of mass relations among members of a given multiplet. For strong interaction physics these appear amazingly well verified and constitute perhaps the most definite support for unitary symmetry. As is well known the baryon octet and decuplet relations are satisfied to within 0.5%, the scalar octet relation to 5%. I shall not go into a detailed derivation of these, but it is important to say a word or two about which relations are better established theoretically than others*. Write the interaction Lagrangian in the form

$$L = L_s + L_{MS} + L_{EM} ,$$

where (i) L_s is the SU_3 symmetric strong interaction for which particles of the same spin and parity form equal mass multiplets. As is well known these can be divided into submultiplets of either I-spin or U-spin, and can be read off most easily from the weight diagrams (see Fig. 2).

(ii) L_{ms} is the medium strong interaction which breaks unitary symmetry but conserves I-spin and hypercharge Y. It produces the splitting between the isotopic submultiplets in a unitary multiplet.

(iii) L_{em} is the electromagnetic interaction which breaks I-spin but conserves U-spin and hence charge Q (which in U-space plays the same role as hypercharge in I-space). It induces the mass splitting between the members of an I-spin multiplet. Since this involves the emission and absorption of a photon, L_{EM} is of order

$$\alpha - e^2 = \frac{1}{137} .$$

Now L_{MS} is a scalar in I-space. Thus in the absence of L_{EM}, but to any order in L_{MS}, all members of an I-spin multiplet have the same mass. Similarly, since L_{EM} is a scalar in U-space, in the absence of L_{MS} but to any order in L_{EM}, all members of a U-spin multiplet have the same mass. The general mass relations we are seeking are therefore those which are satisfied both by conservation of I-spin alone or by U-spin alone. These relations can be obtained very simply from the weight diagrams.

Consider any parallelogram of points in a weight diagram as illustrated in Fig. 2. If we neglect L_{EM}, to all orders in L_{MS}

$$\begin{aligned} m(1) &= m(2) \\ m(3) &= m(4) \end{aligned} \qquad (1)$$

*The remarks that follow have been made (to my knowledge) by Okun, Ahiezer and Schwinger (papers submitted to this Conference) and in the critical form I have presented them, by P.T.Matthews and G. Feldman (Imperial College preprint 1964)

If we neglect L_{MS}, to all orders in L_{EM}

$$m\,(1) = m(4)$$
$$m\,(2) = m(3) \qquad (2)$$

Clearly to all orders in L_{MS} and to all orders in L_{EM} (but neglecting terms $L_{EM} \times L_{MS}$) the one relation which replaces (1) and (2) is

$$m\,(1) - m(2) + m(3) - m(4) = 0.$$

This is called parallelogram law by Matthews and Feldman. They justify the neglect of $L_{EM} \times L_{MS}$ terms by remarking that experimentally, L_{MS} appears to be 1/10 and L_{EM} ~1/137. The interference terms therefore are at least of order 10^{-3}. The parallelogram law should therefore provide some of the most accurate tests for unitary symmetry.

To take an example, for the decuplet we get from its three parallelograms,

$$N^{*-} - N^{*0} + Y^{*0} - Y^{*-} = 0 \qquad (3)$$
$$N^{*0} - N^{*+} + Y^{*+} - Y^{*0} = 0 \qquad (4)$$
$$Y^{*-} - Y^{*0} + \Xi^{*0} - \Xi^{*-} = 0 \qquad (5)$$

At the Conference we have heard some evidence showing that (3) and (4) are verified.

For the baryon octet, there are two particles which appear in the centre, Λ and Σ. The parallelogram law therefore includes a term containing the transition mass $m(\Lambda,\Sigma)$ which arises from remarking that in U-space the scalar combination is $\Lambda_u = \frac{1}{2}(\sqrt{3}\,\Sigma^0 + \Lambda^0)$ while $\Sigma_u = \frac{1}{2}(\Sigma^0 - \sqrt{3}\Lambda^0)$ is the third component of the vector with n and Ξ^0 as the other two components. For the octet there are altogether two parallelogram relations:

$$\left.\begin{array}{l} n - p + \Sigma^+ - \Sigma^0 + \sqrt{3}(\Sigma\Lambda) = 0 \\ \Xi^0 - \Xi^- + \Sigma^- - \Sigma^0 + \sqrt{3}(\Sigma\Lambda) = 0 \end{array}\right. \qquad (6)$$

Eliminating the transition mass we get Coleman-Glashow 6-mass relation

$$n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0 \qquad (7)$$

Including as it does L_{MS} to all orders, and with no restriction on the precise form of L_{MS}, this is the best established theoretical relation in the subject. It should provide one of the severest tests for unitary symmetry. With present evidence the relation in fact appears verified to within experimental accuracy [2].

So far we have retained in the computation of physical masses terms like

$$M = M_o + \sum_{r,s} [(L_{MS})^r + (L_{EM})^s]$$

but neglected the interference terms like $(L_{MS} \times L_{EM})^n$. It is crucial to remark that no special form for L_{MS} was assumed apart from the general requirement that it conserves I-spin and hypercharge. The verification of (7) was therefore essentially a verification of the statement that the photon is a scalar in U-space (and that N, Σ, Λ, Ξ etc., form multiplets in U-space). We now for the first time assume a special form for L_{MS} which asserts that L_{MS} transforms as the I = 0, Y = 0 component of an octet.

In U-space this implies that

$$L_{MS} = \tfrac{1}{2}U_s - \sqrt{\tfrac{3}{2}}\, U_3 \tag{8}$$

To the first order in L_{MS} (and all orders in L_{EM}) we therefore get for the mass-splittings an equal-spacing rule in U-space.

For the decuplet this reads

$$N^{*-} - Y^{*-} = Y^{*-} - \Xi^{*-} = \Xi^{*-} - \Omega^- \tag{9}$$

For the baryon octet

$$n - \Sigma_u = \Sigma_u - \Xi^o \tag{11}$$

or equivalently

$$2(n + \Xi^o) = 3\Lambda + \Sigma^o - 2\sqrt{3}(\Sigma\Lambda).$$

Eliminating $(\Sigma\Lambda)$ transition mass from (6) and (11), we get the mean-mass version of Gell-Mann-Okubo formula

$$(n+p) + (\Xi^o + \Xi^-) = 3\Lambda + (\Sigma^+ + \Sigma^- - \Sigma^o) \tag{12}$$

This incorporates L_{EM} to all orders but L_{MS} to only the first. The interference $L_{EM} \times L_{MS}$ term of course is still not taken into account [3].

(c) Model-dependent mass-relations

In addition to these there are two other types of mass-relations which seem experimentally well established. These are:

(1) Mixing relations between "impure" multiplets. An example is Schwinger's highly accurate quadratic relation between $(\text{mass})^2$ of ϕ, ρ, ω and K* particles :-

$$(\phi-\rho)\ (\omega-\rho) = \tfrac{4}{3}(K^*-\rho)\ (\phi-\omega-2K^*) \tag{13}$$

(2) Intra-multiplet Relations

Examples are

$$K^* - \rho = K - \pi \tag{14}$$

or the remarkable equality noted by Coleman and Glashow :

$$a(8) = a(10), \qquad b(8) = b(10) \tag{15}$$

Here a and b are the parameters in the standard Okubo-Gell-Mann formula

$$M = M_o + aY + b(\underset{\sim}{I}^2 - \tfrac{1}{4}Y^2) \tag{16}$$

and a(8), b(8) refer to the octet, and a(10) and b(10) to the decuplet. These relations differ from (3)-(12) in one very important respect. Whereas (3)-(12) are general consequences of group-theoretic considerations, the mixing-relations or the intra-multiplet relations are consequences (at least so far as present derivations go) of specific dynamic models.

(3) Electromagnetic Mass-Differences

The same remark applies to the detailed phenomenological calculation of electro-magnetic mass-differences (which agree with experiment to 0,5 MeV) carried out (and reported at the Conference) by Coleman and Glashow [4] and by Marshak. I shall take up these model-dependent mass-relations later.

(d) Magnetic Moments of Baryons

The next, not so precise, test for SU_3 comes from comparison of baryon magnetic moments. If photon is scalar in U-space and the symmetry-breaking term L_{MS} is neglected, from the weight diagrams we get :

$$\begin{aligned} \mu_p &= \mu_{\Sigma+} , \\ \mu_{\Xi^-} &= \mu_{\Sigma^-} ; \\ \mu_n &= \mu_{\Xi^0} = \mu_{\Sigma_U} , \end{aligned} \tag{17}$$

where

$$\mu_{\Sigma_U} = \tfrac{3}{4}\mu_\Lambda + \tfrac{1}{4}\mu_{\Sigma^0} - \tfrac{\sqrt{3}}{2}\mu_{\Sigma\Lambda} .$$

If it is assumed that the electro-magnetic current transforms like

$$J_3 + \frac{1}{\sqrt{3}} J_8 \tag{18}$$

we get the two additional relations

$$\mu_n = 2\mu_\Lambda = -2\mu_{\Xi^0} . \tag{19}$$

The new measurement of μ_Λ reported at this Conference gives

$$\mu_\Lambda = -0,66 \pm 0,35 \qquad \text{(in } \Lambda \text{ magnetons)}.$$

Considering the difficulties of precise measurement, this may possibly be called agreement with theory, at least in the sight of God. I shall however comment on the precise significance of the result later.

(e) Decay Widths

Next to the (essentially diagonal) mass or magnetic-moment matrix elements, it is simplest to include the effect of symmetry-breaking terms for the decay amplitudes $F(p_1^2, p_2^2, p_3^2)$

$$A \rightarrow B+C$$

$$(p_1) \quad (p_2, p_3)$$

This has been done for the decuplet decays $10 \rightarrow 8+8$ by V. Gupta and V. Singh and by C. Becchi, E. Eberle, G. Morpurgo. These authors find 7 relations between 12 possible amplitudes. These relations resemble Gell-Mann-Okubo rules and have the form

$$2(N^* \rightarrow N\pi) + 2(\Xi^* \rightarrow \Xi\pi) = 3(Y_1^* \rightarrow \Lambda_\pi) + (Y_1^* \rightarrow \Sigma\pi) \tag{20}$$

Assuming that one may neglect the effect of relative mass differences in $F(m_1^2, m_2^2, m_3^2)$, an experimental comparison for the left and the right sides of (20) gives

$$7.58 \pm 83 (\mathrm{BeV})^{-1} = 7.44 \pm 0.83\ (\mathrm{BeV})^{-1}$$

(f) Cross-Section Relations

The ultimate test of unitary symmetry, of course, is the equality of reaction cross-section. Now the reaction amplitude for a two-body process

$$A + B \rightarrow C + D$$

$$p_1, p_2 \ ; \ p_3, p_4$$

is a function of six invariants $F(p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_1 - p_3)^2)$.

To incorporate the effects of the symmetry-breaking interaction is an art still in its infancy. To see the drastic change which even a partial inclusion of symmetry breaking can produce, consider the example of reactions

$$\begin{aligned}
&(a) \quad \pi^- + p \rightarrow N^{*-} + \pi^+ \\
&(b) \quad K^- + p \rightarrow Y_1^{*-} + \pi^+ \\
&(c) \quad \pi^- + p \rightarrow Y_1^{*-} + K^+ \\
&(d) \quad K^- + p \rightarrow \Xi^{*-} + K^+
\end{aligned} \tag{21}$$

reported by Snow.

Using U-space methods, one can show that in the pure SU_3 limit

$$M_a/\sqrt{3} = -M_b = M_c = -M_d \tag{22}$$

As Fig.3 shows, this is far from the experimental case. Inclusion of symmetry breaking to the first order leaves just one relation between amplitudes

$$M_a + M_b = M_c + M_d \tag{23}$$

Noting that (experimentally) $M_b \simeq M_d \approx 0$ this amounts to checking if $M_a \approx M_c$, which from the data presented is not unreasonable.

I am here taking a highly optimistic viewpoint about predictions of unitary symmetry regarding cross-sections equalities. The hope is that when one has learnt how to include symmetry breaking properly, the tests would be more meaningful. The blunt truth is that if these were the only possible tests of SU_3, one would never, at any rate at the present stage of the subject, have given much credence to unitary symmetry.

Summarizing

Unitary symmetry has a small but impressive list of successes, mainly in predicting mass relations. The successes are more impressive than one has any right to expect. It has however no outright failures. This is partly because, unlike other symmetry proposals, unitary symmetry does not forbid strong reactions otherwise allowed by I-spin and hypercharge conservation. The failures of unitary symmetry can reasonably be ascribed to our inability to include symmetry breaking except to the first order.

II. Composite Models and Unitary Triplets

The relative success of group theoretical models for unitary symmetry naturally leads one to examine its basic group-structure more closely. And here one immediately meets with a deep puzzle. Why does nature not employ the basic triplet representations of the unitary group, when from these elementary (spinor) representations one could compositely construct the tensor representations 1, 8, 10... etc., to which the physical particles seem to belong? In other words, why has the Sakata model failed? Or has it indeed failed? Could it be that the fundamental Sakata-like triplets do exist, not as the physical entities p, n and Λ, but in a different guise? During the last year a number of proposals have been made to employ the triplet representations. I shall examine some of the models. Even though some of these claim to be dynamical in intent, the dynamics are of the most rudimentary character, the essential content being group-theoretic.

(A) The Revolutionary Quark Model

The most economical of all composite models is the Quark (or the Ace) model. Given the Bose multiplets, 1 and 8, and the Fermi multiplets, 1, 8 and 10, find the one unit from which these multiplets, can be composed? The unique answer* is a spin $\frac{1}{2}$ triplet $A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$ where A_1, A_2, A_3, carry baryon number $B = 1/3$ and with the other quantum numbers**

	I_3	Y	$Q = I + Y/2$
A_1	$\frac{1}{2}$	1/3	2/3
A_2	$-\frac{1}{2}$	1/3	-1/3
A_3	0	-2/3	-1/3

This is essentially the Sakata triplet with a charge displacement -1/3. Clearly the world of the quarks, A_1, A_2, A_3, if such exotic objects exist, is a world orthoganal to the world we are used to, in the sense that such particles could be created only in pairs from the known particles. Quarks would constitute a new type of stable matter.

(B) Conservative Triplet Models

For most other models the fractional value of electric charge is too high a price to pay for the economy of having a single triplet. All known particles can be formed as composites either from two triplets [5] or from one Fermi

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* This is because $3 \times 3^* = 1 + 8$
$3 \times 3 \times 3 = 1 + 8 + 8 + 10$

** In terms of the generators of SU_3, Y is defined as $Y = J_8/\sqrt{3}$.
Thus $Q = J_3 + \frac{1}{\sqrt{3}} J_8$ universally for all hadrons as well as for Quarks.

triplet and a neutral singlet. Now all triplets with integral charge fall basically into 2 categories:

(A) Sakata-like triplets

$$S = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \qquad \begin{array}{c} Q \\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{array} \qquad \begin{array}{l} Q = I_3 + \frac{1}{2} Y + \frac{1}{2} C \\ C = 1 \end{array}$$

(B) Lepton-like triplets

$$L = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \qquad \begin{array}{c} Q \\ \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \end{array} \qquad \begin{array}{l} Q = I_3 + \frac{1}{2}Y + \frac{1}{3} C \\ C = -2 \end{array}$$

For both types of triplets, the integral charge requirement forces us to introduce a new quantum number C. This quantum number has been given different names by different people: "addivitive triality" by the Rochester group, "peculiarity" at CERN, "supercharge" by Okun. Personally, I prefer the name given to it by Glashow and Bjorken. They call it the "charm"*. Note that $C \neq \langle\langle Q \rangle\rangle$. For ordinary matter $C = 0$.

Following a classification given by Van Hove and Gell-Mann, one may consider three distinct alternatives:

(1) The new quantum number C is absolutely conserved.

Since for ordinary matter $C = 0$, the triplets then are a new type of stable matter. This case is as exciting as the case of Quarks. Lee and Gürsey have speculated that it is this type of matter which constitutes the substance of the mysterious (Quasi Stellar) Radio Sources.

(2) C is violated by weak interactions.

In this case C is closely parallel to hypercharge so far as its conservation is concerned and the triplets carry a new form of strangeness. The charmed (or charming) particles can only be produced in pairs strongly, though they can decay singly into normal matter. On account of its analogy with leptons, an attractive example of a composite theory is of all (hadronic) matter being built up from an L-type Fermi triplet along with a neutral singlet $\begin{pmatrix} 0 \\ -1 \\ -1 \\ \hline 0 \end{pmatrix}$ fermion.

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* C defined above is 2/3 times the number defined by Glashow & Bjorken.

(3) C is violated semi-strongly, though $\Delta(Y + \frac{2}{3}C) = 0$ in order that $\Delta Q = 0$, $\Delta I = 0$. The "charming" particles can be created singly - though possibly less copiously than those without charm. This model can be realized either*

(a) through one S-type triplet + a neutral singlet

(b) or two triplets as in the models of Schwinger, Van Hove, Lee Gürsey and Nauenberg.

(C) Dynamical Predictions

Consider briefly some of the specific predictions of the various models. Their predictions are as a rule very similar.

(1) The Quark Model

Assuming that Quarks are fairly heavy, Zweig has built up a dynamical model of their binding to give the mass relations between the known SU_3 multiplets. The model has the following characteristics:

1. The medium-strong symmetry breaking is introduced by assuming that the masses of the basic Quarks are different,

$$m_{A_3} > m_{A_1} = m_{A_2} \tag{24}$$

i.e.

$$L_{MS} = (m_{A_3} - m_{A_1})\, A_3^+ A_3$$

2. Since $M_{A_1} = M_{A_2}$, it immediately follows that the residual symmetry is of the U_2 group. This directly leads therefore to the following solutions of the ω, ϕ, ρ mixing problem: the physical particles (the eigen-states) have the transformation properties (corresponding to representations of U_2):

$$\begin{aligned} \rho &= \frac{1}{\sqrt{2}}\left(A_1^+ A_1 - A_2^+ A_2\right) \\ \omega &= \frac{1}{\sqrt{2}}\left(A_1^+ A_1 + A_2^+ A_2\right) \\ \phi &= A_3^+ A_3 \, . \end{aligned} \tag{25}$$

The squared masses satisfy the two relations**

$$\begin{aligned} &1) \quad \rho = \omega \\ &2) \quad 2\phi + \rho + \omega = 4K^* \end{aligned} \tag{26}$$

---

* For an L-type triplet + a singlet one could not simultaneously conserve Y and violate C.

** Relation (2) is a consequence of 1st order symmetry breaking. (1) and (2) have also been derived by Lee, Gürsey & Nauenberg.

3) Assuming that both 0^- and 1^- bosons bind from a Quark and an anti-Quark, and assuming that the binding is independent of spin, one gets the relation

$$K^* - \rho = m_{A_3} - m_{A_1} = K - \pi. \tag{14a}$$

I hope this is not an unfair sample of the type of dynamical argument used in this and other composite models.

It is a type of argument calculated to send a self-respecting S-matrix theorist into fits of despair - despair because the results seem to have the sanction of nature. The most charitable thing one could say about these calculations is what Dr. Johnson once remarked about a woman's preaching. "A woman preaching, Sir, is like a dog walking on his hind legs; it is not done well, but you are surprised to find it done at all."

(2) Schwinger's Field Theory of Matter

1. Starting with a dynamical analogy between leptonic interactions and strong interactions, Schwinger introduces 2 sets of triplet fields to build compositely all known hadrons.

These are:

one Sakata-like Fermi triplet ψ, B = 1

one Sakata-like Bose triplet V, B = 2

2. The crucial assumption is made that at the most elemental level of dynamic theory, ψ and V transform as representations of two independent unitary groups

$$\psi' = U_1 \psi,$$
$$V' = U_2 V,$$

$U_1 \neq U_2$. We are thus dealing with a $(U_3 \times U_3 = W_3)$ group structure. At this level there are 9 baryons $V\bar{\psi}$ corresponding to a $(3,3)^*$ representation of W_3.

3. Mesons (with the group-structure $\bar{\psi}\psi$ transforming as $U\bar{\psi}\psi U^{-1}$) correspond to a reducible 9-fold (9 = 1 + 8) representation of U_3.

4. There are two symmetry-breaking terms; one is introduced to split the 9-fold of mesons into a singlet and an octet*, the other bypasses the SU_3 structure leading directly from W_3 to U_2. The second interaction $(L_{MS} = \bar{\psi}_3(\bar{\psi}_i V_i))$ is something of a tour de force. It is precisely the unaesthetic feature of

* In effect this is tantamount to giving the meson singlets a base mass different from the octets.

bosons carrying two units of baryonic number which forces on the theory this particular type of symmetry breaking. Note that in the second order $L_{MS} \times L_{MS}$ gives the effective interaction of Zweig type

$$\bar{\psi}_3 \psi_3 (\bar{\psi} V)(\bar{V} \psi).$$

The quadratic mass formula connecting ϕ, ρ, ω and K^* $(\text{mass})^2$ mentioned earlier follows directly as the lowest order perturbation arising from the interplay of the two symmetry-breaking terms. Some further features of Schwinger's model are the following:

(a) The decuplet $3/2^+$ is part of a 45-component multiplet which under symmetry breaking splits as 45 = 8 + 10 + 27. Glashow and Kleitman (Phys. Lett...) have given arguments for believing that the 27-fold multiplet is likely to be fairly massive (2 BeV or more).

(b) If the symmetry-breaking terms are ignored, a number of processes are forbidden (compare the Sakata model). For example

$$\pi^+ p \not\to K^+ \Sigma^+$$

$$K^- n \not\to K^0 \Xi^0$$

$$p \bar{p} \not\to \bar{K}^0 K^0$$

Since the symmetry-breaking terms are assumed to be quantitatively enormous, this forbiddenness is perhaps irrelevant.*

(3) Groups of Rank Higher than 2

Given a new quantum number (C), a group theorist will immediately rush off to his copy of Dynkin and make an inventory of all groups of rank higher than 2. Recall that the rank of a Lie group gives the number of its commuting generators - and therefore the number of conserved quantities it can accommodate. SU_3 is a group of rank 2; it can accommodate two quantum numbers (I_3 and Y). Fig. 4 shows Dynkin diagrams for some higher rank groups. Of groups of rank 3, the favourite ones are SU_4 and SP_6 (the symplectic group). The number of authors who have considered SU_4 as a possible super-symmetry of nature is legion [6].

The symplectic group has only one set of votaries [7]. The elementary representation of SU_4 is a quartet (an S or an L-type of triplet + a singlet);

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* My personal view is that the most significant part of Schwinger's theory is not so much its dynamical content but the introduction and the insistence upon the wider group-theoretic structure $U_3 x U_3$. I know Schwinger disagrees with me. I shall however return to this topic later.

the corresponding representation for Sp6 has 6 components (one S and one L-type triplet*.

SU_4

As I stated earlier, all SU_4 models fall into 2 categories

SU_4 Mark I

S-type Quartet $\begin{pmatrix} 1 \\ 0 \\ \underline{0} \\ 0 \end{pmatrix}$

This allows for all three alternatives:

Either (1) C - absolute conservation

or (2) C - weak violation

or (3) C - semi-strong violation.

SU_4 Mark II

L-type quartet $\begin{pmatrix} 0 \\ -1 \\ \underline{-1} \\ 0 \end{pmatrix}$

This allows only

Either (1) C - absolute conservation

or (2) C - weak violation.

Some of the SU_4 representations possess the following content:

(1) 0^-, 1^- = adjoint representation, which in terms of SU_3 multiplets decomposes as follows:

$$4 \times 4^* = 1 + 15 = 8 + 3 + 3^* + 1$$

The submultiplets 3 and 3^* carry charm while for the singlet $C = 0$. Clearly this singlet provides a natural place within the group structure for a ninth boson, (ω^0 or the $/\pi\pi\eta/$).

(2) $\frac{1}{2}^+$ fermions could belong to a 20-fold representation which splits as

$$20 = 8 + 6 + 3 + 3^*$$

---

* Notice that the adjoint representation of SU_4 (to which must belong spin one particles) contains 15 components; the adjoint representation of Sp6 is richer and admits of 21 (1^-) entities.

(3) $3/2^+$ belongs to different 20'-fold which would split as

$20' = 8 + 6 + 3 + 3$

The next 2 tables taken from Glashow & Bjorken; and Amati, Bacry, Nuyts, Prentki, illustrate some of the simple mass assignments, assuming that the SU_4 symmetry is broken in a "natural" (Gell-Mann-Okubo-like) manner. (Fig. 5 and 6). Some people do not know when to stop.

(4) Tests for the Existence of Triplets

If the "charmed" triplets do indeed exist, is there some indirect but recognizable effect they would produce which could constitute a test of their existence?

In so far as the chief distinguishing feature of the triplets is the additive term in the Gell-Mann-Nishijima formula ($C \neq 0$)

$$Q = J_3 + \frac{J_8}{\sqrt{3}} + \frac{C}{3} \tag{27}$$

the answer must lie within electromagnetism. Nauenberg & Okun for example have noted that the relation

$$\mu_N = 2\mu_\Lambda \tag{28}$$

no longer holds if $C \neq 0$. (Note that for Quarks, $C = 0$, so that Quarks do not produce any "indirect" electromagnetic effects.)

Now the violation of (28) certainly constitutes a test of the existence of the triplets. But this test has the drawback that the formula (28) is no longer valid (at present to an unpredictable extent) also when the symmetry-breaking L_{MS} terms are included. Thus if $\mu_N - 2\mu_\Lambda \neq 0$, one would not know if this was the result of the presence of triplets or a consequence of the normal symmetry-breaking mechanism.

A better test possibly is provided by the old chestnut,

$$R = \frac{\phi \to \gamma \to \mu^+ + \mu^-}{\omega \to \gamma \to \mu^+ + \mu^-}$$

Let us assume that the physical particles ω and ϕ are mixtures of a pure "singlet" ω_o and an "octet" ϕ_o.

$$\begin{aligned} \phi &= \phi_o \cos\theta_s + \omega_o \sin\theta_s \\ \omega &= -\phi_o \sin\theta_s + \omega_o \cos\theta_s \end{aligned} \tag{29}$$

The angle θ_s can be determined from strong interactions along (e.g. as suggested by Sakurai by using the relation

$$\Gamma_{\varphi \to K+\bar{K}} = \cos^2\theta \, \Gamma_{\varphi^0 \to K+\bar{K}} ,$$

where $\Gamma_{\varphi^0 \to K+\bar{K}}$ is determined from Γ_{φ} and Γ_{K^*}). Now write

$$R = \left| \frac{\cos\theta + X \sin\theta}{-\sin\theta + X \cos\theta} \right|^2 = \cot^2 \theta_{EM} \qquad (30)$$

Clearly if $C = 0$, $X \neq 0$ and $\theta_{EM} \neq \theta_S$. Conversely, if $\theta_{EM} \neq \theta_S$, and if the notions of unitary symmetry are correct, there must exist triplets of integral charge.

If the triplets are very massive, in general X will be small. There are however certain models (e.g. Schwinger's) where irrespective of the mass of the triplets, $\theta_{EM} - \theta_S$ can be as large as 60° in the exact W_3 limit.

Summarizing

The problems raised by the triplet models are highly significant and of the deepest relevance to the future of Physics. The triplets may be stable; they exist either in the form of Quarks or they may carry integral charge. In this case they define a new and a hitherto unsuspected régime of physical phenomena. The significance of this new régime for cosmology has been speculated - it may or may not concern us here to-day. We cannot however fail to be fired by their significance.

III. Group Extensions and Super-Symmetries

I now turn to what I consider some of the most significant contributions to this Conference. This is the elegant study of the group algebras connected with extensions of SU_3. The study itself is not new. It was carried out in 1961 within the context of unitary symmetry by (see ref.1) M. Gell-Mann, A. Salam and J.C. Ward, and, in terms of a four-field Fermi interaction, by R. Marshak and S. Okubo [8]. It has naturally acquired a renewed significance with the emergence of SU_3 [9].

The story starts with what Gell-Mann called F and D couplings and F and D currents. Consider the interaction of pseudoscalar mesons with baryons. Write the conventional 3 x 3 matrix for the baryons.

$$
\mathcal{B} = \begin{pmatrix} \frac{Y^o}{\sqrt{3}} + \frac{\Lambda^o}{\sqrt{6}} + \frac{\Sigma^o}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{Y^o}{\sqrt{3}} + \frac{\Lambda^o}{\sqrt{6}} - \frac{\Sigma^o}{\sqrt{2}} & n \\ \Xi^- & -\Xi^o & \frac{Y^o}{\sqrt{3}} - \frac{2\Lambda^o}{\sqrt{6}} \end{pmatrix} \qquad (31)
$$

and similarly for the mesons M.

The three-field interaction can be written either in the form :

$$\mathrm{Tr.}\ B^+ B M$$

or in the form :

$$\mathrm{Tr.}\ B^+ B M$$

These are the only two ways of multiplying three matrices within the trace operation. Now with Gell-Mann one can define the symmetric and anti-symmetric combinations of the above two couplings as follows :

$$\mathrm{Tr.}\ B^+ (B M + M B) = \mathrm{Tr.}\ B^+ \{B, M\};$$ the so-called D-coupling

and $$\mathrm{Tr.}\ B^+ (B M - M B) = \mathrm{Tr.}\ B^+ [B, M];$$ the so-called F-coupling

One of the important fundamental parameters in the theory is F/D ratio.

One way to remember the distinction of F and D is to remark that for F couplings there is no $\Sigma \rightarrow \Lambda + \pi$ transition, and for pure D case there is no $\Sigma \rightarrow \Sigma + \pi$ transition. The vector couplings of ρ, K*, φ and ω are conventionally assumed as pure F. For π-mesons, however, hyperfragment binding clearly calls for non-zero $D(g_{\pi\Lambda\Sigma} \neq 0)$. The dynamical calculations of Martin and Wali and others go even further and show that not only must the D-coupling exist for pseudoscalar mesons, they must predominate $(F/D \approx 1/3)$. The same story seems to repeat itself for weak interactions, where the γ_5-currents (axial-vectors) appear predominantly D, the vector currents are F.

The question arises: within the unitary symmetry scheme, what is the origin of F and D couplings; or if we consider vector particles - what is the origin of two types of distinct currents F and D?

The unique answer lies in the group extension $SU_3 \times SU_3$. Consider the two unitary triplets A and B transform as

$$A^1 = U_1 A$$
$$B^1 = U_2^* B$$

If the known 9-folds, e.g. the baryon nonets, are formed as

$$= AB^T = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} (B_1 \ B_2 \ B_3)$$

transforms as

$$\psi' = U_1 \psi U_2^{-1} \qquad (32)$$

As stressed earlier (in connection with Schwinger's field theory of matter), ψ belongs to the $(3,3^*)$ representation of SW_3, provided $U_1 \neq U_2$. If $U_1 = U_2$, i.e.,

$$\psi' = U_1 \psi U_1^{-1}$$

we are dealing with the (reducible) 9-fold representation of SU_3 alone.

Now there is a standard procedure for generating conserved currents - the so-called gauge procedure corresponding to any given transformation. In its essentials, the procedure is to write the transformation concerned infinitesimally, e.g., write

$$U_1 = 1 + iX \quad , \quad U_2 = 1 + iY$$

where X and Y are hermitian 3 x 3 matrices.

The transformation (32) reads

$$\psi' = (1+iX)\psi(1-iY) = \psi + i(X\psi - \psi Y).$$

Likewise $$\partial_\mu \psi' = \partial_\mu \psi + i \ (X_\mu \psi - Y_\mu).$$

The free energy $\bar{\psi}\gamma_\mu \partial_\mu \psi$ therefore transforms to

$$\bar{\psi}\gamma_\mu \partial_\mu \psi + i \ \bar{\psi}\gamma_\mu \ (X_\mu \psi - \psi Y_\mu)$$

The extra terms generated by this procedure represents the coupling of spin one objects X_μ and Y_μ with the baryon-currents.
Rewriting these we get

$$\bar{\psi}\gamma_\mu X_\mu \psi - \bar{\psi}\gamma_\mu \psi Y_\mu = \tfrac{1}{2}\bar{\psi}\gamma_\mu [X_\mu + Y_\mu, \psi] + \tfrac{1}{2}\bar{\psi}\gamma_\mu \{X_\mu - Y_\mu, \psi\} =$$
$$= F^V + D^V .$$

Starting therefore with (32) we see that we have generated naturally both F^V as well as D^V currents. If we had specialized to the case $U_1 = U_2$ (i.e. $X = Y$), we would have generated only the algebra corresponding to F^V alone.

It is easy to check that the commutation relations of F^V and D^V are as follows:

$$[F_i, F_j] = if_{ijk}F_k$$
$$[F_i, D_j] = if_{ijk}D_k$$
$$[D_i, D_j] = if_{ijk}F_k$$

Now so far we have had no axial vector currents (or the corresponding ps.γ_5 coupling). But we know these exist; in fact that for the γ_5 case they predominate. To generate these the standard procedure once again is to consider in the zero baryon mass limit, the two-component entities

$$\psi_L = \frac{1+\gamma_5}{2}\psi,$$
$$\psi_R = \frac{1-\gamma_5}{2}\psi,$$
$$\psi = \psi_L + \psi_R.$$

One can now make 4 independent transformations

$$B_L' = U_1 \, B_L \, U_2^{-1}$$

$$B_R' = U_3 \, B_R \, U_4^{-1}$$

Clearly one will now generate 4 types of currents*.

$$F^V$$
$$F^A$$
$$D^V$$
$$D^A$$

In its widest form then and assuming that possibly corresponding to these currents there also might exist physical particles, we may have a total of sixteen 1^- and sixteen 1^+ particles.

Now it is possible (and indeed quite probable) that nature does not use the generous freedom afforded by all the possibilities listed above. An

---

* Note that each set contains 8 conserved currents (conserved in the limit $m_o \rightarrow 0$) so that the overall algebra generated by these 32 currents, with the commutation relations

$$[F_i^{V,A}, F_j^{V,A}] = if_{ijk}F_k^V, \qquad [F_i^{V,A}, F_j^{A,V}] = if_{ijk}F_k^A,$$
$$[F_i^{V,A}, D_j^{V,A}] = if_{ijk}D_k^V, \qquad [F_i^{V,A}, D_j^{A,V}] = if_{ijk}D_k^A,$$
$$[D_i^{V,A}, D_j^{V,A}] = if_{ijk}F_k^V, \qquad [D_i^{V,A}, D_j^{A,V}] = if_{ijk}F_k^A.$$

in the algebra of $SU_3 \times SU_3 \times SU_3$.

There are of course in addition 4 SU_3 singlets making a total of 36 entities reminiscent of SU_6.

attractive restricted special case is the following :

$$B_L = U_1 B_L U_2^{-1}$$
$$B_R = U_2 B_R U_1^{-1} \qquad \text{(i.e. take } U_1 = U_4,\ U_3 = U_2\text{)}$$

In this case there are only F^V and D^A currents.

(1) It is an attractive hypothesis (forced upon us by the existence of D currents and their dominance for the γ_5 case) that there is possibly in nature a super-symmetry corresponding to $SU_3 \times SU_3$. The baryon 9-fold belongs to the representation,

$$(3,3^*)_L + (3^*,3)_R$$

(2) The symmetry exists in the limit $m_o = 0$

(3) There may exist a normal octet of 1^- (C = −1) and a normal (C = +1) octet of 1^+ particles, corresponding to $(1,8)\pm(8,1)$ representations.

(4) In addition to these 1^- and 1^+ particles, there may exist (0^+) and (0^-) mesons. These spin zero entities may belong either (like baryons) to the nonet representation $(3,3^*)\pm(3,3^*)$ (C = +1) or like vector particles correspond to $(1,8)\pm(8,1)$ (with C = 1 for 0^- and −1 for 0^+).

(5) What happens to the symmetry when the baryon mass is turned on?

Gell-Mann computing in the lowest order shows that the baryon nonet then splits into a singlet and an octet, with

$$m_{\text{singlet}} = 2m_{\text{octet}}$$

Interpreting the negative mass particle as one with opposite parity, the first prediction of this higher symmetry group is that the 9th baryon may be twice as heavy as the octet but with spin-parity $\frac{1}{2}^-$.

(6) For the scalar and pseudoscalar meson $(\text{mass})^2$ spectrum, Gell-Mann and Marshak et al. obtain for the $(3,3^*)\pm(3^*,3)$ the following results :

$\mu^2+2\Delta$ ———————— $0^-(1)$

$\mu^2+\Delta$ ———————— $0^+(8)$

μ^2 ....

$\mu^2-\Delta$ ———————— $0^-(8)$

$\mu^2-2\Delta$ ———————— $0^+(1)$

With the inclusion of Gell-Mann-Okubo type of symmetry breaking, and assuming that the now ubiquitous K = 730 MeV is indeed the "strange" number of the 0^+ octet, one predicts [10] :

$$K^1 = K = 730 \text{ Mev (input)}$$

$\pi' \quad = \quad$ 560 Mev ($G = -1$; decay modes $2\pi+\gamma$ to order α, $2\pi+2\gamma$, 2γ, 2π to order α^2)

$\eta' \quad = \quad$ 770 Mev

If the 0^+ and 1^+ objects exist, where are they?*

To my mind, this is one of the deeper mysteries of the situation. Personally I have no doubt in my mind the extended algebra $SU_3 \times SU_3$ has something to do with nature. That corresponding to each component of the algebra, there exists a physical particle which is an extrapolation from the existence of 1^- and 0^- particles. It is possible that this extrapolation is not wholly warranted, at least in the simple form it has been used so far.

IV. Dynamical Models

In so far as dynamical models are relevant to my material, these fall into two classes:

First are the models which start conservatively with an 8-fold of baryons and mesons and using the methods of S-matrix theory (and assuming trilinear couplings) predict the existence of the 10-fold (or lack of binding for some other multiplets). This of course is good Physics. Its crowning achievement is in the work of Wali and Warnock who show that a broken octet (broken in the sense that the masses satisfy the G-M-O mass relations) leads dynamically to a broken decuplet (again broken in the sense of equal mass spacing).

The next degree of sophistication is to seek to establish the existence of the starting 8-fold itself from the reciprocal self-consistence of a Bootstrap. This would provide a "dynamical origin" for the observed symmetries. The still higher sophistication is to look for a spontaneous breakdown of the symmetry within the stability and the over-riding uniqueness postulates of the Bootstrap approach.

---

* There appears to be a fair sprinkling of 1^+ entities all over the mass spectrum. There are enough possible suspects even to make an octet and a singlet (e.g. $\phi' = 1415$, $\omega' = 980$, $K^* = 1320$, $\rho' = 1220$ Mev seem to satisfy $2\phi'+\rho'+\omega' = 4K^*$, $K' - \pi' \approx K^* - \rho \approx K'^* - \rho'$) but the multiplet appears to possess the wrong C-parity, $C = 1$).

The Bootstrap idea - traced recently by Lovelace at Imperial College to Baron Munchhausen* - is an extremely attractive idea. It is basically the idea that the physical universe is unique and the uniqueness demand coupled with analyticity and unitarity is sufficient to predict the observed features of the Universe including its symmetries.

I think both in theology and cosmology, from the very nature of these disciplines, one always looks at the problem of the structure of the Universe in this light. For elementary particle theory, however, this type of thinking is new, deep and potent. I believe among natural philosophers Voltaire was the first to voice something similar to this. Voltaire attributed to Leibnitz the principle that we live in the best of all possible worlds. The modern theoretical physicist seems to go beyond Leibnitz in asserting that we live not only in the best of all possible worlds - but in the only possible world. In lighter moments I sometimes wonder if the principle does not have the ring of the comforting thought with which Dr. Pangloss made life worth enduring for honest Candide. This was on the occasion of the famous Lisbon earthquake when 30,000 persons lost their lives. Let me quote from the famous Doctor. "Candide, there is no effect without cause and in this best of all possible worlds everything is necessarily for the best - a volcano at Lisbon, it could not be anywhere else, for it is impossible that things be not where they are - and all is well."

Let me summarize the situation as I see it.

I do not know who first used the word <u>strange particles</u> to characterize some of the most exciting objects one has discovered in Physics. Perhaps the smallest measure of change that has come over the subject during the last year is that strange particles are strange no more - and that the strangeness quantum number is as little or as much strange as isotopic spin or electric charge.

There is a suspicion that there might exist still higher symmetry - with SU_3 as possibly an important link in the symmetry chain. There may be a new quantum number; it may be connected with the existence of triplets of integral charge. These triplets (the Sakatons in a completely new guise) at their most exciting, may be a new form of Matter. It is a prospect before which imagination reels.

---

* The Baron lifted himself out of a swamp by his bootstraps. History narrates that the Baron's achievement was not appreciated by his contemporaries.

But with all this optimism there is also mixed a feeling of awe - awe at the magnitude of our ignorance.

We do not know what dynamical mechanism gives this tremendous stability to the mass calculations. Is it that there are very heavy basic triplets, with masses of several BeV binding fiercely and defining a mass scale before which the baryon mass differences are but a small perturbation? Notwithstanding the heroic efforts of the bootstrap physicist, we do not quite yet understand where the origin of the symmetries lies. Or is it that this question is as futile as asking why space-time has dimensionality four? The discovery of the symmetry group of strong interactions was an achievement but when one thinks of the problems that remain one wonders if this was perhaps not the last of the relatively simpler problems. Somehow perhaps the harder tasks remain - the deeper, the more challenging understandings have yet to come.

Before I close I have one more debt to pay. In 1962, V. Weisskopf summed up the spirit of the CERN Conference with Pyramids (Fig. 7).

During 1963 the major item of news was the unfortunate demise of the Regge Pole Model. The next slide presented at the Stanford Conference captures the spirit of 1963 (Fig. 8). Since then the Pyramids have become something of a tradition.

The apprehensive fears of 1964 - perhaps somewhat exaggerated - are shown in the next slide (Fig. 9).

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1/ 1. J. Ne'eman, Nucl. Phys. 26, 222 (1961)

2. M. Gell-Mann, Phys. Rev. 125, 1067 (1962)

The unitary group was first used in elementary particle physics by S. Ogawa, Y. Ohnuki, M. Ikeda (Progr. Theor. Phys. 22, 715, (1959) and Y. Yamaguchi (Progr. Theor. Phys. Suppl. (1960) 11. These authors correctly predicted the completion of the 0^- multiplet (with η^0) though they followed Sakata in assigning baryons to a 3-fold representation. A. Salam and J.C. Ward (Nuovo Cim 10, 20, 419 (1961)) predicted existence of octets of (1^-) and (1^+)) gauge particles. The importance of spin one multiplets lies in the fact that the gauge particles must correspond to the regular representation of a given symmetry group and therefore provide its "invariant" signature (in contrast to any other representations). The 8-fold way assigns not only 0^- and 1^- particles to octet representation but also baryons $\frac{1}{2}^+$.

2/ R. Dalitz (Phys. Lett. 5, 53 (1963)) has used (6) directly to compute the transition mass ($\Lambda\Sigma$) and compared the result with that obtained from a study of the binding of mirror hypernuclei He^4 and H^4. The agreement is not unsatisfactory.

3/ Unlike (7) there probably is no tremendous gain in writing the mean-mass form (12). This is because the neglect of $(L_{MS})^2$, $(L_{MS})^3$, is more serious than taking into account the highest orders of L_{EM}.

4/ As noted by Okubo, the inclusion of L_{MS} to the first order unfortunately leaves only the equi-distance rule in U-space,

This is independent of assumption (18). The other relations (and in particular $\mu_n = 2\mu_\Lambda$) no longer hold.

5/ Two triplets models have been considered by the following:

1. H.Bacry, J.Nuyts, L.Van Hove, preprint CERN (1964). (two Fermi triplets)
2. J.Schwinger, preprint; F.Gürsey, T.Lee and M.Nauenberg, preprint (one Fermi and one Bose triplet)

6/
1. S. L.Glashow and Pjorken. Preprint. Phys.Lett.11, 255 (1964)
2. V.V.Vladimirsky, SU_4-symmetry. preprint.
3. P.Tarjanne and V.L.Teplitz, Phys.Rev.Lett. II, 447 (1963)
4. W.Krolikowski, Nucl.Phys. (to be published).
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6. Y.Hara. Phys.Rev. 134, B 701 (1964)
7. Z.Maki and Y.Ohnuki. Quartet Scheme for Elementary Particles. Preprint (1964)
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9. D.Amati, H.Bacry, J.Nuyts and J.Prentki. SU_4 and Strong Interactions. Phys.Lett. 11. N2, 100 (1964)

7/ H.Bacry, J.Nuyts, L.Van Hove, preprint CERN.

A theory based on Sp6 has certain similarities with Schwinger's theory. In particular these authors also derive the ρ, ϕ, ω, K^* quadratic formula:

$$(\omega-\rho)(\phi-\rho) = \frac{4}{3}(K^*-\rho)(\phi+\omega-2K^*).$$

8/ R.Marshak and S.Okubo, Nuovo Cim. 19, 1226 (1961)

9/ Work on this topic was reported at the Conference by:

1. Y.Nambu and P.G.O.Freund
2. M. Gell-Mann
3. R.Marshak, N.Mukunda and S.Okubo
4. A.Salam and J.C.Ward

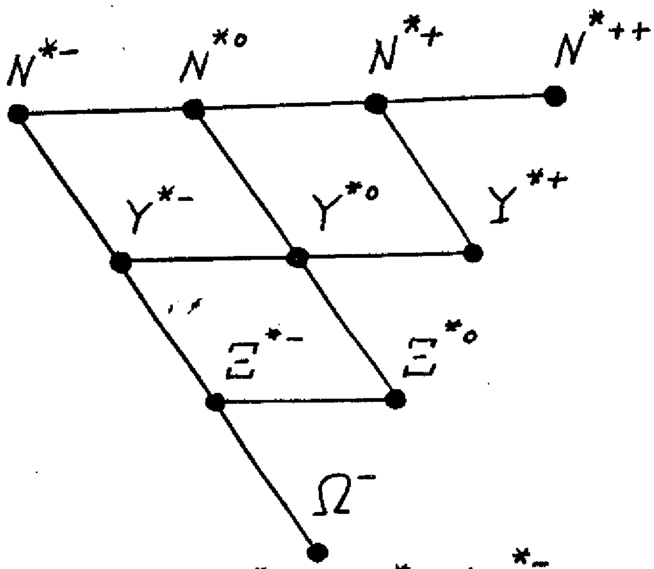
10/ For the C = -1 (abnormal) case, Marshak et al give the following values:

$$K^1 = 688 \text{ Mev}$$
$$\eta' = 630 \text{ Mev}$$
$$\pi' = 837 \text{ Mev}$$

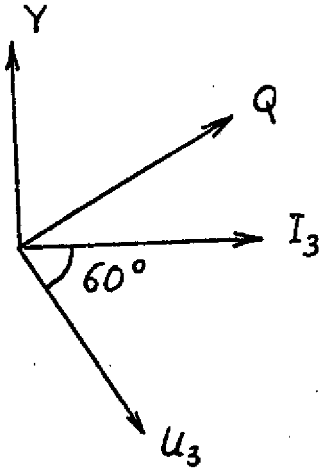
The "abnormal" case was first considered by Nambu and Sakurai (Phys. Rev. Letters 11, 42 (1963) who showed that the production and decay rates of a C = -1 octet are highly suppressed.

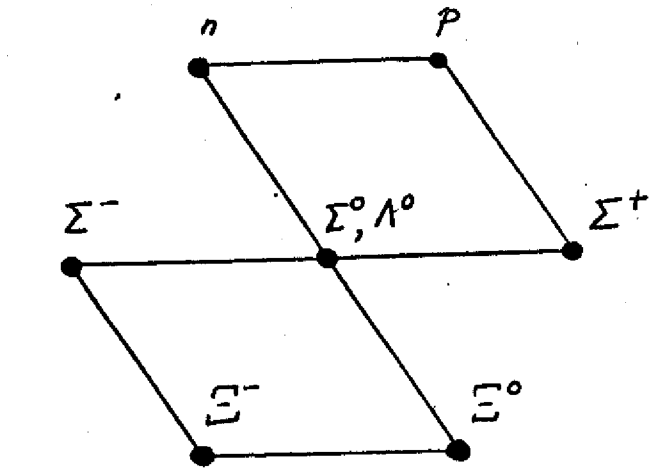
Figure 1

1. M. Ademollo, R. Gatto, G. Preparata
2. D. Amati, H. Bacry, J. Nuyts, J. Prentki
3. A. I. Akhiezer, M. P. Rekalo
4. H. Bacry, J. Nuyts, L. Van Hove
5. A. I. Baz
6. A. M. Baldin, A. A. Komar
7. C. Becchi, E. Eberle, G. Morpurgo
8. S. Coleman, R. Socolow, S. L. Glashow, H. E. Schnitzer
9. B. Diu, H. R. Rubinstein, J. L. Basdevant
10. K. Fujii, K. Iwata
11. Y. Fujii, M. Ichimura, K. Yazaki
12. E. V. Gedalin, O. V. Kancheli, L. B. Laperashvilli
13. M. Homa, K. Matumoto, S. Tanaka
14. B. L. Ioffe, I. Yu. Kobsarev, I. Ya. Pomeranchuk
15. O. S. Ivanitskaya, A. E. Levashov
16. A. Kotanski, K. Zalewski
17. A. J. MacFarlane, N. Mukunda, E. C. G. Sudarshan
18. Z. Maki, Y. Ohnuki
19. R. E. Marshak, S. Okubo
20. M. E.Mayer, H. J. Schnitzer, E. C. C. Sudarshan, R. Acharya, M. Y. Han
21. S. Meshkov, G. A. Snow, G. B. Yodh
22. S. Nakamura
23. S. Nakamura
24. Y. Nambu, P. G. O. Freund
25. V. I. Ogievetski
26. V. I. Ogievetski, I. V. Polubarinov
27. L. O. Raifeartaigh, T. S. Santhanam, E. C. G. Sudarshan
28. A. Ramakrishnan
29. J. J. Sakurai
30. R. F. Sawyer
31. V. M. Shekhter
32. V. V. Vladimirski
33. J. P. Vigier, F. Hallwachs, P. Hillion, M. Flato
34. K. C. Wali, R. L. Wornock
35. J. H. Woitaszek, R. E. Marshak, Riez-ud-din
36. C. Zemach
37. M. Gell-Mann
38. J. Schwinger

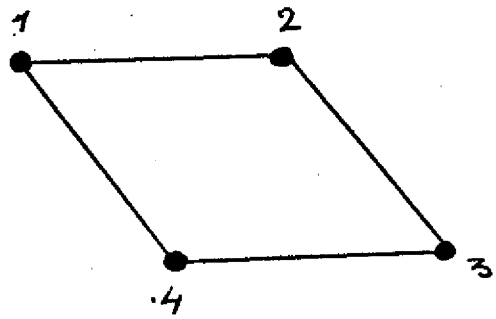


$$N^{*-} - N^{*0} + Y^{*0} - Y^{*-} = 0$$



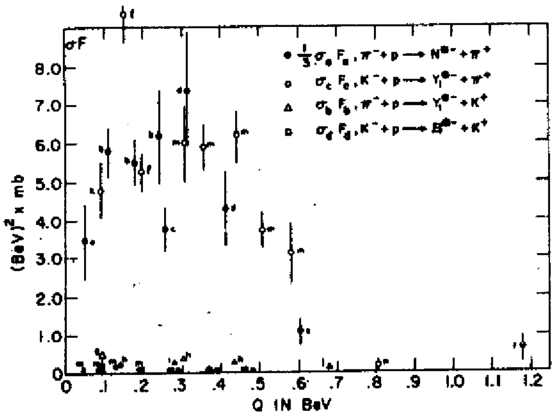


$$\left.\begin{array}{l} n - p + \Sigma^+ - \Sigma^0 + \sqrt{3}\,(\Sigma\Lambda) = 0 \\ \Xi^0 - \Xi^- + \Sigma^- - \Sigma^0 + \sqrt{3}\,(\Sigma\Lambda) = 0 \end{array}\right\} \Rightarrow n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0$$



$$m(1) - m(2) + m(3) - m(4) = 0$$

Fig. 2



σF
$\frac{1}{3}\sigma_a F_a$, $\pi^- + p \longrightarrow N^{*-} + \pi^+$
$\sigma_c F_c$, $K^- + p \longrightarrow Y_1^{*-} + \pi^+$
$\sigma_b F_b$, $\pi^- + p \longrightarrow Y_1^{*-} + K^+$
$\sigma_d F_d$, $K^- + p \longrightarrow \Xi^{*-} + K^+$
(BeV)^2 x mb
Q IN BeV

Fig. 3

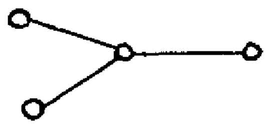
GROUP	DYNKIN DIAGRAM	NUMBER OF ELEMENTARY REPRESENTATION	RANK (Number of conserved quantities)	NUMBER OF VECTOR PARTICLES
U_3		2	2	8
U_4		2	3	15
Sp_6		1	3	21
D_4 (O_8)		3	4	28

Fig. 4

15 Pseudoscalar Mesons.	Y .	C .	I .	15 Vector Mesons
η (550)	0	0	0	ω (790)
χ (950)	0	0	0	ϕ (1020)
π (140)	0	0	1	ρ (750)
K (500)	1	0	1/2	K^* (890)
D_p (760)	0	1	1/2	D_v (770)
S_p (900)	1	1	0	S^+_v (940)

Weak Decay Modes

$$
\begin{array}{lll}
0^- & \left.\begin{array}{l} D_p \longrightarrow \bar{K}\pi \\ S^+_p \longrightarrow \eta\pi \end{array}\right\} & 10^{-12}\,\text{s.} \\
1^- & \left.\begin{array}{l} D^+ \longrightarrow \pi\pi \\ D^0 \longrightarrow \pi\pi\pi,\ \pi\pi,\ \bar{K}\pi \\ S^+ \longrightarrow \pi\pi,\ K\pi \end{array}\right\} & 10^{-14}\,\text{s.}
\end{array}
$$

Fig. 5

B $\frac{3}{2}^+$ resonances (equal spacing law)				
	decuplet	sextet	triplet	singlet
T=0	Ω^- 1672	$\Omega^{-\prime}$ 1950	$\Omega^{-\prime\prime}$ 2228	$\Omega^{-\prime\prime\prime}$ 2506
T=$\frac{1}{2}$	Ξ^*1527	$\Xi^{*\prime}$ 1805	$\Xi^{*\prime\prime}$ 2083	
T=1	Y_1^*1382	$Y_1^{*\prime}$ 1660		
T=$\frac{3}{2}$	N*1237			
These numbers have been computed by considering Y_1^* (1660) as the peculiar $Y_1^{*\prime}$ particle				

Fig. 6

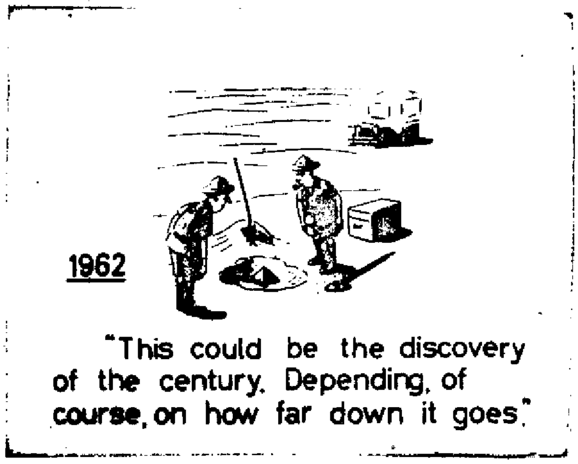


Fig. 7

Fig. 8

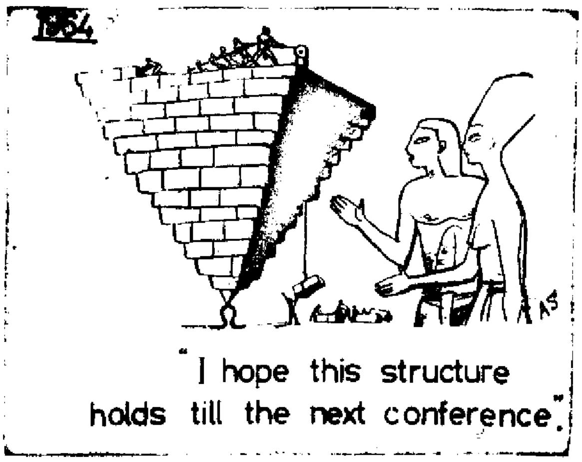


Fig. 9