PELTIER EFFECT OF SUPERCONDUCTOR-SEMICONDUCTOR
MESOSCOPIC DEVICE

Arafa H. Aly
Physics Department, Faculty of Science, Cairo University, Beni-Suef, Egypt
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

Adel H. Phillips
Engineering Physics & Mathematics Department, Faculty of Engineering,
Ain-Shams University, Abbasia, Cairo, Egypt.

Abstract

The heat current density is derived for superconductor- Semiconductor-Superconductor mesoscopic device by using WKB method. A numerical calculation is performed for the heat current at different barrier strength and bias voltages. Our results show that such mesoscopic device could be used as local electron gas thermometers.

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1 Author for all correspondence. E-mail: arafa16@yahoo.com
Introduction:

Modern device fabrication techniques have made it possible to construct tunnel junction devices on the submicron level [1]. Such mesoscopic devices are the next step in the evaluation of small devices whose primary objectives are faster characteristic device times and lower dissipation. New effects rise in this mesoscopic domain as a result of the quantum mechanical phase of the electron, as well as the discrete nature of the electronic charge [2]. One of the effects is the generation of heat current [3] when the electric current in the normal metal-insulator-superconductor (N-I-S) flows. This principle can be applied to the refrigeration of electrons in the normal metal [4]. The mechanism of the heat transfer in the NIS contacts is the same as that of the well-known Peltier effect in metal-semiconductor contacts [5].

In the present paper, we shall deduce an explicit expression for the heat current of mesoscopic junction modelized as superconductor(S) - semiconductor (Sm)-supercondutr(S), sandwiched type junction. Such junction has an advantage that the concentration of carriers in semiconductor can be varied and monitored. So, we can study the effect of barrier strength variation on the heat current. This study is made for the first time for such S-Sm-S junctions.

II. Theoretical Treatment:

Mesoscopic device, in this paper, can be modelized as a semiconductor heterostructure sandwiched between two superconducting leads. The sandwiched part has dimension smaller than the electron phase coherence length as well as the elastic means free path. The superconductor energy gap, $\Delta$, is constant in space up to the S-Sm boundary and is equal to its equilibrium value inside the superconductor [6]. The transport properties of S-Sm contact are influenced by two scattering processes [6,7,8] namely, normal tunneling and Andreev reflection processes. The heat current, $J$, [9] flowing thorough the junction is given by:

$$ J = \frac{1}{A \pi h} \int dE(E - eV)[f^+(E) - f^-(E)] $$

where $A$ is the cross-sectional area of the interface, $h$ is the reduced Planck's constant, $e$ is the electronic charge, $V$, is the bias voltage . In Equation (1) $f^\prime(E)$ is the equilibrium Fermi-Dirac distribution function shifted by $eV$ and is given by Equation (2)

$$ f^+(E) = f(E - eV)........ ........ ........ ........ ........ (2) $$

While the energy distribution $f(E)$ [9] of quasiparticles due to the Andreev and normal reflections at S-Sm interface is given by:

$$ f^-(E) = A(E)[1 - f^+(E)] + B(E)f^+(E) + [1 - A(E) - B(E)]f(E) .............. (3) $$

where $B(E)$ and $A(E)$ are the probabilities of normal reflection and Andreev reflection from S-Sm interface as functions of the quasiparticle energy E. The expression for A(E), is given by [9,10]:

$$ A(E) = \frac{1}{1 + B(E)} $$

$$ B(E) = \frac{\tanh(\frac{E}{2kT})}{1 + \tanh(\frac{E}{2kT})} $$

The expression of the equilibrium Fermi-Dirac distribution function is:

$$ f(E) = \frac{1}{1 + \exp(\frac{E}{kT})} $$
\[ A(E) = \frac{\Delta^2}{4Z^2(\Delta^2 - E^2)} \quad \text{For} \quad E < \Delta, \quad (4a) \]

\[ A(E) = \frac{\Delta^2}{4Z^2(E^2 - \Delta^2)} \quad \text{For} \quad E > \Delta, \quad (4b) \]

In Eqs. (4a,b), the parameter Z represents the barrier strength at the S-Sm interface which is expressed in terms of the Schottky barrier height, \( V_b \), as [10].

\[ Z = mV_b / \hbar^2 P_o \]

where \( P_o = \sqrt{\frac{2mE_F}{\hbar^2}} \)

in which \( E_F \) is the Fermi energy.

Now, the heat current, \( J \), can be obtained explicitly by substituting eqs.(2,3) and using the expression for \( A(E) \) (eq.4) we get:

\[
J = \left( \frac{1}{A\hbar \pi} \right) \left[ (E - eV)[1 + \frac{\Delta^2}{4Z^2(\Delta^2 - E^2)}]\left\{ \frac{1}{1 + \exp\left( \frac{E - eV - E_F}{k_BT} \right)} - \frac{\Delta^2}{4Z^2(\Delta^2 - E^2)} \right\} \right] \int [1 - \frac{1}{1 + \exp\left( \frac{-E + eV + E_F}{k_BT} \right)}] - [1 - \frac{\Delta^2}{4Z^2(\Delta^2 - E^2)}]\left\{ \frac{1}{1 + \exp\left( \frac{E - eV - E_F}{k_BT} \right)} \right\} \, dE \quad (6)
\]

Performing the above integral (Eq.6) we get an expression for the heat current, \( J \), as:
\[ J = \Delta^2 \left\{ \ln |\Delta^2 - E^2| + \frac{eV}{\Delta} \tanh^{-1} \left( \frac{E}{\Delta} \right) \right. \\
+ \exp \left( \frac{eV + E_f}{k_{B}T} \right) \left\{ \exp \left( \frac{\Delta}{k_{B}T} \right) \left( -\frac{eV}{2} - \frac{eV}{2\sqrt{5}} \right) (0.5772 + \ln \left( \frac{\Delta}{k_{B}T} + E \right) + 0.2 \left( \frac{\Delta}{k_{B}T} + E \right) \exp \left( -\frac{\Delta}{k_{B}T} \right) \right) \\
+ \left( -\frac{eV}{2} - \frac{eV}{2\sqrt{5}} \right) (0.5772 + \ln \left( \frac{\Delta}{k_{B}T} + E \right) + 0.2 \left( \frac{\Delta}{k_{B}T} + E \right) \exp \left( -\frac{\Delta}{k_{B}T} \right) \right) \\
\left. \left\{ \frac{0.5 \ln |\Delta^2 - E^2|}{\Delta^2} - \frac{eV}{\Delta^2} \tanh^{-1} \left( \frac{E}{\Delta} \right) - \exp \left( \frac{eV + E_f}{k_{B}T} \right) \right\} \left( \frac{\Delta^2}{2(\Delta^2 - E^2)} \right) \right\} \]

\[
\begin{align*}
\exp \left( \frac{-E}{k_{B}T} \right) + [0.5772 + \ln \left( \frac{\Delta}{k_{B}T} + E \right) + 0.2 \left( \frac{\Delta}{k_{B}T} + E \right) \exp \left( -\frac{\Delta}{k_{B}T} \right) ] \\
\left\{ \sqrt{\Delta} \left( \Delta - eV \right) \left( \Delta + 1 \right) + eV \left( \Delta^2 - 1 \right) + \Delta^2 \left( \Delta - eV \right) \right\} 2\Delta^2 \left( \Delta^2 - 1 \right) \left( E + \Delta \right) \\
2\Delta^2 \left( \Delta^2 - 1 \right) k_{B}T \\
\left\{ \sqrt{\Delta} \left( \Delta - eV \right) \left( \Delta + 1 \right) + eV \left( \Delta^2 - 1 \right) + \Delta^2 \left( \Delta - eV \right) \right\} 2\Delta^2 \left( \Delta^2 - 1 \right) \left( E + \Delta \right) \\
\left\{ \sqrt{\Delta} \left( \Delta - eV \right) \left( \Delta + 1 \right) + eV \left( \Delta^2 - 1 \right) + \Delta^2 \left( \Delta - eV \right) \right\} 2\Delta^2 \left( \Delta^2 - 1 \right) k_{B}T \\
\end{align*}
\]
III. Numerical results:
We perform the numerical calculations for the heat current, $J$, eq(7) for the case of GaAs-AlGaAs semiconductor heterostructure and Nb as the superconducting leads. The tunneling through the S-Sm-S junction is treated as a stochastic process, so that we find the value of the Schottky barrier height $V_b$, by using a Monte-Carlo method to be equal to 0.52eV. This value is in a good agreement with those in the literature[7,11]. In this case, we can change the barrier strength, $Z$, at S-Sm interface. Our results are characterized as follows:

Fig.(1) shows the variation of the heat current, $J$, with the temperature, $T$, of the junction. The heat current decreases as the temperature increases. This result has been confirmed by us [7,8,10,12] previously and by many other authors [2,11]. From this result, we can conclude that the junction under investigation operates as a Josephson junction.

Fig.(2) shows the variation of the heat current $J$, with the bias voltage $V_o$, for different values of the barrier strength $Z$, at S-Sm interface. From the figure we notice that the heat current increases as the bias voltage $V_o$ increases and then decreases at a certain value of $V_o$ for a certain value of the barrier strength $Z$. When varying the barrier strength, the variation of the heat current with $V_o$ is the same but the maximum of the heat current $J_{max}$ shifts towards higher values of $V_o$. This can be shown from Fig.(3). When analyzing our results we see that the heat current $J$, in this limit is maximum at temperature $Z$-0.4$\Delta$, where $(\Delta=\Delta_o/K_BT)$, and $\Delta_o$ is the energy gap of Nb-superconductor. From this result the Andreev reflection process at S-Sm interface plays a very important role in the tunneling process through S-Sm-S mesoscopic junction. From Fig.(2) we notice that the heat current, $J$, increases as the barrier strength decreases.

Our results found a good concordant with the results of other authors but for normal metal-insulator-superconductor microcontacts.

IV. Conclusion:
In the present paper, we have obtained an explicit expression for the heat current, $J$, for S-Sm-S mesoscopic junction, for the first time. The heat current was deduced for both normal and Andreev reflection at S-Sm interface and expressed explicitly in terms of the barrier strength at S-Sm interface. Our results found concordant with those in the literature. In conclusion, such results show that such mesoscopic devices could be used as local electron gas thermometers.

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References:
Fig. 1 temperature dependence of the heat current density.

Fig. 2 Bias voltage dependence of the heat current density.
Fig. 3 The variation of bias voltage with maximum heat current density.