In this work we determine theoretically the critical charge density in the system grounded metallic sphere, uniformly charged dielectric plane, in the presence of grounded surfaces, in a more general case. Special attention is payed to the influence of the system geometry in determining the most optimal conditions for obtaining the minimum critical charge density. This is a situation frequently encountered in industrial condition and is important in evaluating the danger of the electrostatic discharges.
1 Introduction

The critical charge density $\sigma_c$ is the surface charge density that corresponds to the critical intensity of the field that provokes a discharge. Its determination is important because in most cases in industry, when relatively not small and non-conductive surfaces are charged, electric fields are produced that are able to create a gas discharge. If the atmosphere is explosive, the situation may become dangerous [1-13].

Theoretically, in a first approximation, we have examined the problem of determining critical charge density, for clean configuration grounded metallic sphere-uniformly charged dielectric plane [14-16]. But in concrete industrial conditions there always exist a ground surface. For this reason, in a second approximation we have determined $\sigma_c$ taking into consideration grounded surface, placed parallelly with charged dielectric plane [17]. But in reality in most cases there does not exist such a parallel configuration. For this reason in this work we will determine $\sigma_c$ in a more general case.

2 Presentation of the problem

The system we have examined [16] has been grounded sphere-uniformly charged dielectric plane (Figure 1) that is needed for the estimation of the danger of electrostatics charges [18-25].

The plane, uniformly charged with density $\sigma$, is rectangular with sides $a$ and $b$, placed respectively in the distance $d$ and $D$ from the grounded surface and grounded sphere with $r$ radius.

The intensity of the electrostatic field in $S$ point reaches its maximum value in the configuration shown in Figure 2 [17], and it is just this value which is of interest for the determination of the critical charge density.

To pass to the more general case, the configuration shown in Figure 3 has to be examined, in which the intensity of electrostatic field in $S$ point has to be determined.

3 Determination of $E_s$ in more general case

The model equivalent scheme (Figure 3) for the determination of $(E_s)_{max}$ is represented in Figure 4, where plane $P'$ and sphere $A'$ are respectively the mirror images of plane $P$ and sphere $A$. The electric field intensity in the $S$ point is:

$$E_s = E_{p,s_1} + E_{p',s_2} + E_{s_1'} + E_{s_2'} + E_{S_1} + E_{S_2} + E_{S_1'} + E_{S_2'}$$

(1)

where $E_{p,s_1}$ and $E_{p',s_2}$ are the fields created respectively by the uniformly charged planes $P$ and $P'$, together with their charges $q_{s_1}$ and $q_{s_2}$, which are the mirror images in sphere $A'$; $E_{s_1'}$ and $E_{s_2'}$ are the fields created by the charges $q_{s_1'}$ and $q_{s_2'}$, which are respectively the mirror images of the charged planes $P'$ and $P$ in sphere $A'$; $E_{S_1}$ and $E_{S_2}$ are the fields created by the charges $q_{S_1}$ and $q_{S_2}$. 

2
and \( q^*_s \), which are respectively the mirror images of \( q_s \) and \( q_s' \) in the sphere \( A \); \( \vec{E}_{s_1'} \) and \( \vec{E}_{s_2'} \) are the fields created by the charges \( q^*_s \) and \( q^*_s \), mirror images of \( q_s' \) and \( q_s'' \), in the sphere \( A \).

The sphere \( A \) represents the measuring electrode, and taking into consideration the experimental industrial conditions it can be assumed

\[ r \ll a, b, d \tag{2} \]

In this non-symmetric case, then according \([16, 26, 28]\), and also considering specific properties of the general case, that we are looking for, (Figure 5), we can write:

\[
\begin{align*}
(E_{p,s_1})_x & = -\frac{\partial \varphi}{\partial x} = -\frac{\sigma}{2\pi \varepsilon_0} \left( \frac{2\pi + K}{\pi} \right) \\
(E_{p,s_2})_x & = -\frac{\partial \varphi}{\partial x} = -\frac{\sigma}{4\pi \varepsilon_0 r \cos^2 2\alpha} \left( -4dK_5 \cos \alpha + K_6 + \frac{1}{2} K_7 \right) \\
(E_{p,s_2})_y & = -\frac{\partial \varphi}{\partial y} = -\frac{\sigma}{4\pi \varepsilon_0 r \sin^2 2\alpha} \left( -4dK_10 \cos \alpha + \frac{K_{11}}{2} \sin 2\alpha + K_{12} \right) \\
(E_{s_1'})_x & = -\frac{\partial \varphi}{\partial x} = -\frac{\sigma r}{4\pi \varepsilon_0 d \cos^2 \alpha} \left( \frac{K_{13}}{2} ight) \\
(E_{s_1'})_y & = -\frac{\partial \varphi}{\partial y} = \frac{6r}{4\pi \varepsilon_0} \cdot \frac{1}{2d^2 \sin^2 \alpha} \left( -rK_{16} + \frac{K_{17}}{4d \sin \alpha} + \frac{K_{18}}{2} \right) \\
(E_{s_2'})_x & = -\frac{\partial \varphi}{\partial x} = \frac{6r}{4\pi \varepsilon_0 d \cos \alpha} \left[ \frac{1}{4d \cos \alpha} \left( \frac{K_7}{2} + K_{19} \right) - K_5 \right]
\end{align*}
\]

where:

\[
\begin{align*}
K & = a \ln \frac{b + \sqrt{a^2 + b^2}}{a} + b \ln \frac{a + \sqrt{a^2 + b^2}}{b} \\
K_1 & = b - 4d \sin \alpha \\
K_2 & = b + 4d \sin \alpha \\
K_3 & = \sqrt{a^2 + b^2 + 16d^2 + 8bd \sin \alpha} \\
K_4 & = \sqrt{a^2 + b^2 + 16d^2 - 8bd \sin \alpha} \\
K_5 & = \arctan \frac{aK_1}{4dK_3 \cos \alpha} + \arctan \frac{aK_2}{4dK_4 \cos \alpha} \\
K_6 & = a \ln \frac{K_3 + K_1}{K_4 - K_2} \\
K_7 & = K_1 \ln \frac{K_3 + a}{K_3 - a} + K_2 \ln \frac{K_4 + a}{K_4 - a} \\
K_8 & = \sqrt{K_3^2 \sin^2 2\alpha + 4r(K_1 \sin 2\alpha + r)} \\
K_9 & = \sqrt{K_4^2 \sin^2 2\alpha - 4r(K_2 \sin 2\alpha - r)} \\
K_{10} & = \arctan \frac{a(K_2 \sin 2\alpha + 2r)}{4dK_9 \cos \alpha} + \arctan \frac{a(K_1 \sin 2\alpha + 2r)}{4dK_8 \cos \alpha} \\
K_{11} & = (K_1 \sin 2\alpha + 4r) \cdot \ln \left( \frac{K_8 + a \sin 2\alpha}{K_8 - a \sin 2\alpha} + (K_2 \sin 2\alpha + 4r) \cdot \right) \ln \left( \frac{K_9 + a \sin 2\alpha}{K_9 - a \sin 2\alpha} \right) \\
K_{12} & = a \ln \frac{K_0 + (K_2 \sin 2\alpha - 2r)}{K_8 - (K_1 \sin 2\alpha + 2r)} 
\end{align*}
\]
\[ K_{13} = K - 2r \arctg \frac{ab}{2r\sqrt{a^2 + b^2}} \]

\[ K_{14} = \sqrt{(a^2 + b^2)d^2 \sin^2 \alpha - r^2(2bd \sin \alpha - r^2)} \]

\[ K_{15} = \sqrt{(a^2 + b^2)d^2 \sin^2 \alpha + r^2(2bd \sin \alpha + r^2)} \]

\[ K_{16} = \arctg \frac{a(bd \sin \alpha - r^2)}{2rK_{14}} + \arctg \frac{a(bd \sin \alpha + r^2)}{2rK_{15}} \]

\[ K_{17} = (bd \sin \alpha - 2r^2) \cdot \ln \left( \frac{2K_{14} + ad \sin \alpha}{2K_{14} - ad \sin \alpha} \right) + (bd \sin \alpha + 2r^2) \cdot \ln \left( \frac{2K_{15} + ad \sin \alpha}{2K_{15} - ad \sin \alpha} \right) \]

\[ K_{18} = a \ln \frac{2K_{14} + (bd \sin \alpha - 2r^2)}{2K_{15} - (bd \sin \alpha + 2r^2)} \]

\[ K_{19} = a \ln \frac{K_4 + K_2}{K_3 - K_1} \]

We have not shown \((E_{s'})_{y_0}\) because it is parallel with \(P\) plane and does not influence in determining the critical charges density.

Based on relation (2) the charges \(q_{s1}, q_{s2}, q_{s1}', q_{s2}'\), will be considered as point charges, situated, respectively at the center of the spheres \(A\) and \(A'\).

According to [8, 17, 29–31], and also considering specific property of the general case, that we are looking for, the full mirror image of the plane charge on the sphere is

\[ q' = -r\sigma l \]

where in the symmetric case:

\[ I = (I_3) \quad \begin{array}{c|c|c}
D = r & b_1 = b/2 & b_2 = b/2 \\
D = -r & b_1 = -b/2 & b_2 = b/2 \\
2K_{13} &
\end{array} \]

and in the non symmetric case:

\[ (I_{ns}) \quad \begin{array}{c|c|c}
D = (2\cos \alpha - r) & b_1 = -\left(\frac{b}{2} + 2d \sin \alpha\right) & b_2 = \left(\frac{b}{2} - 2d \sin \alpha\right) \\
D = -(2d \cos \alpha - r) & b_1 = -\left(\frac{b}{2} + 2d \sin \alpha\right) & b_2 = \left(\frac{b}{2} - 2d \sin \alpha\right) \\
-4dK_5 \cos \alpha + K_6 + \frac{K_7}{2} &
\end{array} \]

In accordance with relations (2, 29-31), Figure 4 and Figure 6, we can find out:

\[
\begin{align*}
q_{s1} < 0 &= -2r\sigma \cdot K_{13} \\
q_{s2} > 0 &= r\sigma \left(-4dK_5 \cos \alpha + K_6 + \frac{K_7}{2}\right) \\
q_{s1}' > 0 &= 2r\sigma \cdot K_{13} \\
q_{s2}' < 0 &= -r\sigma \left(-4dK_5 \cos \alpha + K_6 + \frac{K_7}{2}\right) \\
\end{align*}
\]
To determine the critical charge density, first, we propose to consider the component of total field $E_{\text{pingul}}$ with plane $P$, beginning from the center 0 of sphere $A$ (Figure 7).

In accordance with relations (2-28, 34-37) and Figure 7 finally the component of total field $E_{\text{pingul}}$ with plane $P$, $(E_s)_n$ can be found from the relation

$$E_{s_1}^+ = \frac{\sigma}{4\pi \varepsilon_0} \cdot \frac{r^2}{4d^3} \cdot K_{13}$$

$$E_{s_2}^+ = \frac{\sigma}{4\pi \varepsilon_0} \cdot \frac{r^2}{8d^3} \left(-\frac{4dK_5 \cos \alpha + K_6 + \frac{K_7}{2}}{2}\right)$$

$$E_{s_1}^- = \frac{\sigma}{4\pi \varepsilon_0} \cdot \frac{K_{13}}{d}$$

$$E_{s_2}^- = \frac{\sigma}{4\pi \varepsilon_0} \cdot \frac{1}{2d} \left(-\frac{4dK_5 \cos \alpha + K_6 + \frac{K_7}{2}}{2}\right)$$

4 Determination of critical charge density

To determine the critical charge density, first, we propose to consider the component of total field $E_{\text{pingul}}$ with plane $P$, $(E_s)_n$ from the center 0 of sphere $A$ (Figure 7).

In accordance with relations (2-28, 34-37) and Figure 7 finally the component of total field $E_{\text{pingul}}$ with plane $P$, $(E_s)_n$ can be found from the relation

$$(E_s)_n = -\frac{\sigma}{4\pi \varepsilon_0} \left(4\pi + \frac{A}{r} + \frac{B}{d} + \frac{C}{d^2} + \frac{F}{d^3} + \frac{G}{d^4}\right)$$

where

$$A = 2K - \frac{1}{\cos 2\alpha} \left(K_6 + \frac{K_7}{2} - 4dK_5 \cos \alpha\right) - \frac{1}{\sin 2\alpha} \left(\frac{K_{11}}{2} \sin^2 \alpha + K_8 - 4dK_{10} \cos \alpha\right)$$

$$B = \frac{rK_5}{2} \cos \alpha + \frac{rK_{13} \cos 2\alpha}{2} + \frac{r \cos \alpha}{\sin \alpha} \left(\frac{K_{18}}{2} - rK_{16}\right) - \frac{r \cos \alpha}{4 \cos^2 \alpha} \left(\frac{K_7}{2} + K_9\right)$$

$$C = \frac{2 \cos^2 \alpha}{r} + \frac{r \cos \alpha}{\sin \alpha} \left(\frac{K_{18}}{2} - rK_{16}\right) - \frac{r \cos \alpha}{4 \cos^2 \alpha} \left(\frac{K_7}{2} + K_9\right)$$

$$D = \frac{2 \cos^2 \alpha}{r}$$

$$E = \frac{r \cos \alpha}{4 \sin^2 \alpha} \cdot K_{17}$$

$$F = \frac{r \cos \alpha}{8} \left[K_{13} - \frac{1}{2} \left(K_6 - 4dK_5 \cos \alpha + \frac{K_7}{2}\right)\right]$$

Secondly, to determine the critical charge density, we must use the condition presented in [16,17]. Normally the sphere radius is taken to be $r = 10^{-2} m$, because different authors [23, 32] think that in general when $r > 10^{-2} m$ the situation will be dangerous for the beginning of the discharge. For determining $(E_s)_n$ we shall use Bower’s method [16, 23] according to which for an ionizing tension of ambient air $V = 12.5 V$, for average path of electron $\lambda = 3.7 \times 10^{-7} m$, and the sphere’s radius $r = 10^{-2} m$, discharge at the point of the sphere is reached when
After the determination of \(|(E_s)_n| = E_0\) and of \(r\), according to (38) at last we can write:

\[
\sigma_c = \frac{4\pi\varepsilon_0 E_0}{f(a, b, d, r, \alpha)}
\]

(40)

where \(\sigma_c\) is the critical charge density, while

\[
f(a, b, d, r, \alpha) = 4\pi + \frac{A}{r} + \frac{B}{d} + \frac{C}{d^2} + \frac{F}{d^3} + \frac{G}{d^4}
\]

(41)

is the function of the geometrical parameters of the system.

5 Discussion

The critical charge density \(\sigma_c\) is a function of six parameters

\[
\sigma_c = f_1(a, b, d, r, \alpha, E)
\]

(42)

among which five of them determine the geometry of the system, while the intensity \(E_0\) represents the Bower’s condition for the discharge creation. Among these six parameters, two of them, will definitely be fixed at the parameter’s values \(r = 10^{-2}m; E_0 = 4.56 \times 10^6\)\(\text{v/m}\). While the parameters \(a, b, d\) are considered variable subjected to condition (2) and the parameter \(\alpha\) that varies in interval \((0.1 - 0.7)\text{rad}\) and \((0.87 - 1.48)\text{rad}\).

To verify the accuracy of equation (40) it is important to study the function \(\sigma_c = f(d)\).

In Figure 8 we have graphically presented \(\sigma_c = f(d)\) for parameter values \(a = 1.5m; b = 2m; \alpha = \frac{\pi}{6}\). As \(d\) increases, \(\sigma_c\) decreases and asymptotically tends to the critical charge density of the pure system grounded metallic sphere-uniformly charged dielectric plane, as \(d\) goes to infinity, \(\sigma_c^\infty = 8.14 \times 10^{-7}\frac{\text{v}}{\text{m}^2}\).

In the determination of the critical charge density the influence of the \(\alpha\) parameter is very important. For this reason in Figure 9 we have graphically presented \(\sigma_c = f(d)\) for parameter values \(a = 1.5m; b = 2m; \alpha = \frac{\pi}{3}\). As \(d\) increases, \(\sigma_c\) increases and asymptotically tends to the critical charge density of the pure system grounded metallic sphere-uniformly charged dielectric plane, as \(d\) goes to infinity, \(\sigma_c^\infty = 8.14 \times 10^{-7}\frac{\text{v}}{\text{m}^2}\).

The critical density of the clear system can be found analytically from relation (40)

\[
\sigma_c^\infty = \lim_{d \to \infty} \frac{4\pi\varepsilon_0 E_0}{f(a, b, d, r, \alpha)} = \frac{2\varepsilon_0 E_0}{K}\frac{\sigma}{2}\]

(43)

because

\[
\lim_{d \to \infty} f(a, b, d, r, \alpha) = 4\pi + \frac{2K}{r}
\]

(44)

Equation (43) is in full agreement with our previous results [14–16], a fact which definitely points out the accuracy of equation (40).

From the general equation (40) we can also pass into the special case when the grounded surfaces are placed parallely to the charged plane. In this case:

\[
(\sigma_c)_{\alpha=0} = \lim_{\alpha \to 0} \sigma_c = \frac{4\pi\varepsilon_0 E_0}{f_0(a, b, d, r, \alpha)}
\]

(45)
where

\[
\begin{align*}
A_0 &= 2K + 8dK_{21} - K_{22} \\
B_0 &= K_{13} + 2(2d + r)K_{21} - \frac{K_{22}}{2} \\
C_0 &= \frac{r}{4}(2K_{13} - K_{22}) \\
G_0 &= \frac{r^3}{8} [K_{13} - \frac{1}{2}(K_{22} - 8dK_{21})] \\
K_{20} &= \sqrt{a^2 + b^2 + 16d^2} \\
K_{21} &= \arctan \frac{ab}{4dK_{20}} \\
K_{22} &= a \sqrt{\frac{K_{20} + b}{K_{20} - b}} + b \frac{\ell n}{\ell n} \frac{K_{20} + a}{K_{20} - a}
\end{align*}
\] (47)

Equation (45) is a general one and includes our result for the critical charge density as a special case, as in a previous paper [17] on \(\alpha = 0\). This fact not only points once more to the general character of equation (40), but verifies its exactness.

The graph (Figure 8) shows that \(\sigma_c\) increases as \(d\) decreases. The physical explanation of this phenomena is the following: as \(d\) decreases, the field created by the system at point \(S\) decreases, so that the critical charge density must increase, in order to fulfil the Bower’s condition for the start of the discharge. There is a different situation in the graph (Figure 9), in which \(\sigma_c\) increases with the increase of \(d\). The physical explanation of this phenomena is the following: as \(d\) increases, the field created by the system at point \(S\) decreases, so that the critical charge density must increase, in order to fulfil the Bower’s condition for the start of the discharge.

The study of the dependence of critical charge density from the angle \(\alpha\) is very interesting. Its obvious influence is also seen in the graphs presented in Figure 8 and Figure 9. In Figure 10 we have graphically presented \(\sigma_c = f(\alpha)\) for parameter values \(a = 1.5m\); \(b = 2m\); \(d = 8m\). When \(\alpha\) varies in the interval \((0, 1) - (0.7)rad\), the minimal value of the \(\sigma_c\) is \(8.9 \times 10^{-7}c/m^2\) and is achieved at \(\alpha = 0.4rad\). While in Figure 11 we have graphically presented \(\sigma_c = f(\alpha)\) for parameter values \(a = 1.5m\); \(b = 2m\); \(d = 8m\). Figure 11 shows that the variation of \(\sigma_c\) from \(\alpha\), in the interval \((0.87 - 1.48)rad\), has another character. In this way \(\sigma_c\) reaches values even smaller than \(8.9 \times 10^{-7}c/m^2\). This fact is explained by the change of the electrostatic field intensity in the point \(S\), \([(E_x)_n]\), where the major role is played by the sinusoidal and cosinusoidal components, respectively \(\alpha = (E_{pl_x})_{y_1}, (E_{pi_x})_{y_i}\) and \((E_{pl_x})_{y_1}, (E_{pi_x})_{y_2}\). For the determination of \(\sigma_c\) in the point \(\alpha = \frac{\pi}{4}\), we should get the limit of the function presented in (40).

The calculations show:

\[
(\sigma_c)_{\alpha = \frac{\pi}{4}} = \lim_{\alpha \to \frac{\pi}{4}} \sigma_c = \frac{4\pi \varepsilon_0 E_0}{f^*(a, b, d, r, \alpha)}
\] (49)

where

\[
f^*(a, b, d, r, \alpha) = 4\pi + \frac{A^*}{r} + \frac{B}{d^2} + \frac{C^*}{d^3} + \frac{F}{d^4} + \frac{G}{d^5}
\] (50)
\[ A^* = 2K - \frac{1}{\sin 2\alpha} \left( \frac{K_{14}}{2\sin 2\alpha} + K_{12} - 4dK_{10}\cos \alpha \right) \] 
\[ C^* = \frac{r \cos \alpha}{\sin \alpha} \left( \frac{K_{18}}{2} - rK_{16} \right) - \frac{r}{4\cos^2 \alpha} \left( \frac{K_7}{2} + K_{19} \right) \]  
(51)

For the same value of the parameters: \( a = 1.5m; b = 2m; d = 8m, \) we get \((\sigma_c) = 8.39 \cdot 10^{-7} c/m^2\).

Finally, in the abovementioned \( \alpha = \frac{\pi}{4} \) angle intervals, \( \sigma_c \) reaches the minimal value in the "critical zone" \((\frac{\pi}{4} - 1.35 \text{ rad})\) where \( \sigma_c \leq 8.39 \cdot 10^{-7} c/m^2 \).

In Figure 12, we have graphically presented \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; \alpha = \frac{\pi}{4} \). The graph shows that the influence of \( d \) is obvious in the dimensions present in industrial conditions.

At last, in Figure 13 we have graphically presented \( \sigma_c = f(\alpha, d) \) for parameter values \( a = 1.5m; b = 2m; \) and \( \alpha \) varies in the interval \((0,1 - 0,7) \text{ rad}\) while \( d \) varies in the interval \((0 - 10)m\). Figure 13 again shows that the minimal value of \( \sigma_c \) is \( 8.9 \cdot 10^{-7} c/m^2 \) and is achieved at \( \alpha = 0.4 \text{ rad} \).

In conclusion, the advantage of using (40) is that such a relation for a given system allows firstly the computation of \( \sigma_c \) in a more general case, and secondly, varying the parameters \( a, b, d, \alpha \) so that we can give to \( \sigma_c \) the desired value, is possible.

6 Conclusion

A very important problem in industry is determining the limit of critical charge density with avoids discharge on non conductive charged surfaces. In this work we theoretically determine the critical charge density in the system grounded metallic sphere with a uniformly charged dielectric plane, in the presence of grounded surfaces, in the more general case, a situation frequently encountered in industrial condition and is important in evaluating the danger of the electrostatic discharges. Special attention is payed to the influence of the system geometry in determining the most optimal conditions for obtaining the minimum critical charge density.

Our conclusions easily allows one to apply them in special cases, in order to compute the value of \( \sigma_c \) for given systems, and to vary it as a function of the variable parameters which determine the geometry of the system.

Acknowledgments

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References


Figure Captions

Fig. 1 The system grounded sphere-uniformly charged dielectric plane in the presence of the grounded surface.

Fig. 2 The realistic scheme to determine the critical charge density.

Fig. 3 The realistic scheme to determine the critical charge density in the more general case.

Fig. 4 The model equivalent scheme to determine the critical charge density in the more general case.

Fig. 5 The scheme to determine the electrostatic fields \( \vec{E}_{p,s_1}, \vec{E}_{p,s_2}, \vec{E}_{s_1}, \) and \( \vec{E}_{s_2}. \)

Fig. 6 The scheme to determine the electrostatic fields \( \vec{E}_{s_1}^p, \vec{E}_{s_2}^p, \vec{E}_{s_1}^s, \) and \( \vec{E}_{s_2}^s. \)

Fig. 7 The scheme to determine the component of total field pingul with plane \( P, (E_s)_n. \)

Fig. 8 The graphical representation of \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; \alpha = \frac{\pi}{6}. \)

Fig. 9 The graphical representation of \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; \alpha = \frac{\pi}{3}. \)

Fig. 10 The graphical representation of \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; d = 8m; \alpha \rightarrow (0, 1-0,7)rad. \)

Fig. 11 The graphical representation of \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; d = 8m; \alpha \rightarrow (0, 87-1,48)rad. \)

Fig. 12 The graphical representation of \( \sigma_c = f(d) \) for parameter values \( a = 1.5m; b = 2m; \alpha = \frac{\pi}{4}. \)

Fig. 13 The graphical representation of \( \sigma_c = f(\alpha, d) \) for parameter values \( a = 1.5m; b = 2m; \alpha \rightarrow (0, 1-0,7)rad., d \rightarrow (0-10)m. \)
Figure 3
Figure 5
Figure 6
Figure 7
Fig 10

\( \sigma_c (C/m^2) \)

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0.1 0.2 0.3 0.4 0.5 0.6 0.7 \( \alpha (\text{rad}) \)

Fig 11

\( \sigma_c (C/m^2) \)

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0.9 1. 1.1 1.2 1.3 1.4 \( \alpha (\text{rad}) \)
Fig. 12

\[ \sigma_c (\text{C/m}^2) \]

- \( 1.05 \times 10^{-6} \)
- \( 1. \times 10^{-6} \)
- \( 9.5 \times 10^{-7} \)
- \( 9. \times 10^{-7} \)
- \( 8.5 \times 10^{-7} \)

4 8 12 16 20 \( d \) (m)

Fig. 13

\[ \alpha \]

- 0.2
- 0.4
- 0.6

2 4 6 8 \( d \)

\( 1. \times 10^{-6} \)

8 \( \times 10^{-7} \)

0.