United Nations Educational Scientific and Cultural Organization  
and  
International Atomic Energy Agency  
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ANOMALOUS $U(1)_A$ AND ELECTROWEAK SYMMETRY BREAKING

Ilia Gogoladze$^*$  
*E-mail: iliag@ictp.trieste.it  

The Andronikashvili Institute of Physics, Georgian Academy of Sciences,  
Tbilisi 380077, Georgia  
and  
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

Mirian Tsulaia$^†$  
†E-mail: tsulaia@thsun1.jinr.ru  

Bogoliubov Laboratory of Theoretical Physics, JINR,  
Dubna 141980, Russian Federation,  
The Andronikashvili Institute of Physics, Georgian Academy of Sciences,  
Tbilisi 380077, Georgia  
and  
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

We suggest a mechanism for electroweak symmetry breaking in the Supersymmetric Standard Model. Our suggestion is based on the presence of an anomalous $U(1)_A$ gauge symmetry, which naturally arises in the four dimensional superstring theory, and heavily relies on the value of the corresponding Fayet–Illiopoulos $\xi$–term.

MIRAMARE – TRIESTE  
October 2000
1 Introduction

Recently the theories suggesting the fundamental scale of gravitational interaction to be as low as a few TeVs were developed [?]. Within these theories the observed weakness of the Newtonian coupling is due to the existence of large ($\gg \text{TeV}^{-1}$) extra dimensions into which the gravitation flux can spread out. At the distances larger than the typical size of these extra dimensions gravitational potential goes to its standard Newton's law. Moreover, all currently known collider data as well as astrophysical and cosmological observations are barely consistent with the known theoretical construction in the case of two extra dimensions, at least [?].

A number of recent publications consider the origin of electroweak symmetry breaking within the low–scale extra dimension scenario. Several origins of electroweak symmetry breaking and of the Higgs field were suggested. For instance, in [?] Higgs field was considered as a composite particle while in [?] Higgs field was identified with a tree–level massless open string state. In [?] the spontaneous electroweak symmetry breaking by supersymmetric strong dynamics at the TeV scale was studied. Also in [?] the issue of extra dimensions at the TeV scale as a possible origin of electroweak symmetry breaking was suggested.

In the present paper we address the issue of the possibility of electroweak symmetry breaking triggered by the $D_A$–term of anomalous $U(1)_A$ symmetry along with the standard F–terms coming from the $N = 1$ superpotential. To motivate this kind of scenario we can refer to the superstring theory. Until the recent time the compactification of heterotic $E_8 \otimes E_8$ superstring on Calabi–Yau manifolds [?] or alternatively on the orbifolds [?] was considered as the only way to produce the realistic chiral $N = 1$ supergravity theory in four dimensions. However, the recent developments in the string theory has revealed that other superstring theories like the type IIB and I/I' are phenomenologically acceptable as well [?] - [?]. In latter theories as well as in the case of heterotic string [?], the string scale may not be directly related to the Planck scale [?], and, hence, the gauge hierarchy problem is naturally avoided. The compactification of the type IIB (or type I/I') theory using various kind of orientifold constructions leads to the string vacua which contain D–branes. The gauge fields live on the D–branes while gravity still propagates in the bulk. Such kinds of string vacua contains the spectrum of the Supersymmetric Standard Model along with some modulus fields, dilaton, axion, and the fields from the "hidden sector" (see [?] for a review). The $SU(2)_W \otimes U(1)_Y$ gauge symmetry of the Standard Model is enlarged in these kinds of theories by one or several anomalous $U(1)_A$ gauge symmetries. It was shown that the corresponding Fayet–Illiopoulos $D_A$–term can play crucial role in spontaneous supersymmetry breaking [?], as well as in a breaking of some nonanomalous gauge symmetries [?] and in the strong CP–violation problem [?]. Therefore it is interesting to investigate whether the anomalous $D_A$–term has some other impact on the four–dimensional physics namely on electroweak symmetry breaking. We show that at least for the case of one anomalous $D_A$–term such a breaking can take place.
The paper is organized as follows: In Section 2 we recall some basic facts about the origin of anomalous $U(1)_A$ symmetry and corresponding Fayet–Illiopoulos $\xi$–terms in four dimensional superstring theories and we suggest the model where the electroweak symmetry is broken via the $D_A$–term of anomalous $U(1)_A$ symmetry, without presence of mass (mixing) terms for Higgs superfields in four dimensional $(d = 4)$ $N = 1$ lagrangian. In Section 3 we discuss the possible dynamical generation of the $\mu$–term in $d = 4$ $N = 1$ SUGRA due to the condensate of fields from the "hidden sector" having nontrivial charge under $U(1)_A$ symmetry and examine the consequences of the appearance of $\mu$–term in four dimensional lagrangian for the $SU(2)_W \otimes U(1)_Y$ symmetry breaking. Section 4 contains our conclusions and outlook.

2 The model 1

2.1 The origin of anomalous $U(1)_A$ symmetry

The four dimensional field theoretical models obtained after the compactification of either heterotic or type IIB superstring theories contain the anomalous $U(1)_A$ gauge symmetries. For the case of compactification of the heterotic string there exists one anomalous gauge symmetry [?]. The anomalies, being disastrous for the quantum consistency of the theory, are cancel due the four dimensional Green–Schwartz mechanism, analogous to the one taking place in ten dimensional type I and heterotic superstring theories [?], therefore, the four dimensional string theories are consistent as well. Namely, upon the compactification on a six dimensional volume $v$ the $U(1)_A$ anomalies for the heterotic string case are cancel due to the presence in the action of the counter term of the form:

$$\delta_{GS} B \wedge F_{U(1)},$$

along with the coupling term $dB \wedge \omega$, where $\delta_{GS}$ is the Green–Schwartz anomaly cancellation coefficient, being the universal constant of the model; $F_{U(1)}$ is the curvature corresponding to the anomalous gauge field $A_\mu$; $\omega$ is the corresponding Chern–Simons form and $B$ is the four dimensional antisymmetric two–form which gives the pseudoscalar $a$–axion upon the dualisation: $\partial_\mu B_{\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a$. The combination $(s + ia)$ of the axion $a$ and the heterotic string dilaton $s$ forms the lowest component of the chiral superfield $S$, referred to as a complex dilaton superfield. The latter undergoes the transformation $S \rightarrow S + i \frac{1}{2} \delta_{GS} \theta_A$ with the parameter $\theta_A$ under the anomalous $U(1)_A$ gauge group. After developing the gauge invariant kinetic term of the complex dilaton

$$L_s = - \int d^4 \theta \ln (S + \bar{S} - \delta_{GS} V),$$

where $V$ is the corresponding vector superfield, one can see that supersymmetry generates along with (??) the mass term for anomalous $A_\mu$ gauge boson at two–loop level and Fayet–Illiopoulos $\xi$–term at one loop level in string expansion. The value of parameter $\xi$ is given by:

$$\xi = \frac{\delta_{GS}}{4} M_{str}^2 \quad \text{with} \quad \delta_{GS} = \frac{Tr Q_A}{48\pi^2},$$
and $\sqrt{\xi}$ is of order or less of string scale depending on the value of $\text{Tr} Q_A$, where the summation in the trace goes over all charges of superfields which transform nontrivially under the anomalous $U(1)_A$ symmetry. The role of Goldstone boson is played by the axion $a$, which becomes the longitudinal component of anomalous gauge boson giving the mass proportional to $\lambda^2 \delta_{GS} M_{str}^2$ (where $\lambda^2 = 1/(s)$ is the heterotic string coupling constant) and, hence, anomalous $U(1)_A$ gauge symmetry is broken spontaneously.

The situation for the case of compactification to $d = 4$ of the type IIB superstring theory is essentially different since there can be several anomalous $U(1)_A^i$ groups. The quantities $\xi^i$ are proportional to the combination of real parts of the twisted moduli fields coming from NS–NS sector with the model dependent coefficients and vanish in the orbifold limit. However, the anomalous $U(1)_A^i$ gauge bosons obtain the masses of the order of the string scale despite of whether the orbifold singularities are blown up or not \footnote{As we have mentioned in the Introduction the heterotic string scale can be also lowered to the 1 TeV due to the effect of small instantons}. On the other hand $\xi^i$ can be generated non-perturbatively at the tree level in string expansion. The order of parameters $\xi^i$ is undetermined, thus, giving the large room for the model building. In the models considered below we suppose the the order of parameters $\xi$ to be close be the magnitude to the order of type IIB string scale \footnote{As we have mentioned in the Introduction the heterotic string scale can be also lowered to the 1 TeV due to the effect of small instantons} i.e., $\sqrt{\xi} \sim O(10^2)$ GeV in the low scale (TeV) string theory. Again, the four dimensional type IIB superstring theory is anomaly free, but now the cancellations of $U(1)_A^i$ happens due to the generalized Green-Schwartz cancellation mechanism \footnote{As we have mentioned in the Introduction the heterotic string scale can be also lowered to the 1 TeV due to the effect of small instantons} governed by the imagine parts of twisted moduli fields coming from the Ramond-Ramond sector rather than dilaton as it was for the four dimensional heterotic string models.

2.2 The model

The important problems still open in the framework of the MSSM are: what is the origin of $\mu$-term, why (how) a given coupling (or mass) is tiny or zero without any apparent symmetry reasons, why the magnitude of $\mu$-term is of the order of 100 GeV and not of the order of $M_{str}$ which is the fundamental scale in superstring theory.

Therefore we would like to address the question: Is it possible to break SM gauge symmetry without $\mu$-term and without extension of particle spectrum?

In subsequence we shall not specify the string origin of the models considered below. We simply assume that the lagrangian of the supersymmetric Standard Model is enlarged by single anomalous Fayet – Illiopoulos $\xi$- term corresponding to the anomalous $U(1)_A$ gauge symmetry. The $D_A$-term for anomalous $U(1)_A$ has the generic form:

$$D_A = \xi + \sum_i Q_i X_i G_i,$$

where $G_i$ is the derivative of corresponding Kähler potential of $d = 4 \ N = 1$ supergravity with respect to fields $X_i$ having the charges $Q_i$ under the anomalous gauge group. Retaining
the quadratic part of Kähler potential with respect to the matter fields one obtains:

\[ D_A = \xi + \sum Q_i X_i^2. \]  

One can see that it is always possible to define the anomalous \( U(1)_A \) charges of quarks and leptons in the way that all terms of the ordinary MSSM lagrangian are allowed except the \( \mu \)-term. On the other hand we can also use \( U(1)_A \) symmetry for the suppression of \( R \)-parity breaking terms. For example, one of the possible set of anomalous \( Q_i \) charges for one family of quarks and leptons is given in Table 1. Here \( Q_L, U_R, D_R, L, E_R \) correspond to the chiral superfields, which contain ordinary quarks and leptons. \( H_1 \) and \( H_2 \) are MSSM Higgs field.

<table>
<thead>
<tr>
<th>Superfield</th>
<th>( Q_L )</th>
<th>( U_R )</th>
<th>( D_R )</th>
<th>( L )</th>
<th>( E_R )</th>
<th>( H_1 )</th>
<th>( H_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>(-\frac{1}{2}n)</td>
<td>(-\frac{1}{2}n)</td>
<td>(\frac{1}{2}n)</td>
<td>(\frac{1}{2}n)</td>
<td>(-\frac{1}{2}n)</td>
<td>(-\frac{1}{2}n)</td>
<td>(\frac{3}{2}n)</td>
</tr>
</tbody>
</table>

Table 1 The \( U(1)_A \) charges corresponding to the quarks, leptons and Higgs superfields defined up to the multiplication by arbitrary number \( n \).

For our analyses the ratio between anomalous charges and their exact values are not important. To simplify our analyses let us denote corresponding charges of MSSM Higgs (\( H_1 \) and \( H_2 \)) as \( q \) and \( p \) respectively.

According to the previous analyses (possible suppression of \( \mu \)-term) the scalar potential for the neutral components of MSSM Higgs field contains just the contributions from \( D \)-terms corresponding to anomalous and non-anomalous gauge symmetry and is the following:

\[ V = \frac{g^2}{4} (\xi + q|H_1|^2 + p|H_2|^2)^2 + \frac{g_W^2 + g_Y^2}{4}(|H_1|^2 - |H_2|^2)^2, \]  

where \( g_W \), \( g_Y \) and \( g_A \) correspond to gauge coupling constant of \( SU(2)_W \otimes U(1)_Y \otimes U(1)_A \) gauge symmetry. The absolute minimum of the potential (??) is achieved for the following vacuum expectation values of MSSM Higgs fields:

\[ <|H_1|^2> = -\frac{\xi}{q + p}. \]  

Based on the discussion given in the previous subsection the value of \( \sqrt{\xi} \)-terms in the low-scale string theory can be of the order of \( 10^2 \) GeV. So, under the natural assumption that \( p + q \) is of order of the unity we can conclude from eq. (??) that due to the existence of anomalous \( U(1)_A \) symmetry it is possible to break SM gauge symmetry and get the correct values of masses of gauge bosons, Higgs fields, quarks and leptons (as well as of their superpartners) without having the \( \mu \)-term in the theory and without extension of the particle spectrum. However, in the absence of the \( \mu \)-term it is easy to see that one of the charginos and one of the neutralinos from gaugino and higgsino sector become massless.

The simplest way to avoid this problem is to introduce additional superfield \( P \), which is a singlet under the \( SU(2)_W \otimes U(1)_Y \) symmetry and has \( h = -(p + q) \) charge under the anomalous
\( U(1)_A \) symmetry. This allows the existence of the following expression in the superpotential:

\[ W = \frac{k}{\sqrt{2}} H_1 H_2 P. \]

The corresponding potential for \( P \) field and neutral components of MSSM Higgs fields is:

\[ V = \frac{g_A^2}{4} (\xi + q |H_1^0|^2 + p |H_2^0|^2 + h |P|^2)^2 + \frac{g_W^2 + g_Y^2}{4} (|H_1^0|^2 - |H_2^0|^2)^2 + \frac{k^2}{2} (|H_1^0|^2 H_1^0 |H_1^0|^2 + |H_1^0|^2 P|^2 + |H_2^0|^2 P|^2). \]

From the extremum condition we have the following system of linear equations:

\[ (g q^2 + g') |H_1^0|^2 + (p p q - g' + k^2) |H_1^0|^2 + (g h q + \lambda) |P|^2 + g \xi q = 0, \]
\[ (g p q - g' + k^2) |H_1^0|^2 + (g p^2 + g') |H_1^0|^2 + (g h p + k^2) |P|^2 + g \xi p = 0, \]
\[ (g q h + k^2) |H_1^0|^2 + (g p h + k^2) |H_2^0|^2 + g h^2 |P|^2 + g \xi h = 0, \]

where \( g \equiv g_A^2 \) and \( g' \equiv g_W^2 + g_Y^2 \). It is clear that the solution which corresponds to non-zero VEV of \(|H_1^0|^2, |H_2^0|^2 \) and \(|P|^2\) fields is proportional to \( \xi \)-term and, as we noted before, \( \sqrt{\xi} \sim O(10^2 \text{GeV}) \). Hence, in this way it is possible to generate correct VEV for SM Higgs fields. The non-zero VEV of \( P \) field generates effectively the \( \mu \)-term in the superpotential. This is equivalent to the arising of \( \mu \)-term in gaugino-higgsino sector and in this way all neutralinos and charginos become massive.

Thus, in this case we have the self-consistent picture for supersymmetric SM.

3 The model 2

3.1 The possible generation of \( \mu \)-term

The second model that we are consider suggests the \( \mu \)-term to be of some dynamical origin, namely it is a vacuum expectation value of some extra fields living in the "hidden sector". As we will show this is the necessary condition for the realization of mechanism of electroweak symmetry breaking which is based on the existence of anomalous \( U(1)_A \) symmetry.

Let us explain the possibility of the appearance of the \( \mu \)-term in the four dimensional effective action. One way to observe this is to consider the \( d = 4 \) \( N = 1 \) supergravity lagrangian, which can arise after the appropriate compactification of type IIB or heterotic strings. The \( d = 4 \) \( N = 1 \) supergravity lagrangian is a functional of three independent functions \( K, f, f' \) of the dilaton, modulus, visible and hidden sector chiral superfields as well as of the vector superfields which correspond to the gauge interactions. The functions \( K \) and \( f \) contribute to the Kähler potential, while \( f' \) is the gauge kinetic function. The problem of finding the exact expression of these functions in the superstring theory is still open although various string vacua give a certain restrictions on them [7], [7].

For the simplification of our analyses let us discard the dependence of Kähler potential on the modulus fields. The part of the \( d = 4 \) \( N = 1 \) supergravity Kähler potential describing the
interaction of gravity with chiral matter superfields \( y^a \) and the chiral superfields \( z^i \), coming from the hidden sector, is expressed as:

\[
G = K(\Phi, \Phi^+) + \ln |f(\Phi)|^2, \tag{10}
\]

where \( K \) depends on \( \Phi \equiv (y^a, z^i) \) and its conjugate \( \Phi^+ \) and \( f \) is a holomorphic function of \( \Phi \).

As it was shown in [?] one can take the functions \( K(\pi, \pi^+, y, y^+) \) and \( f(\pi, y) \) \((\pi^+ \equiv z^i/M_{Pl})\) to be of the following form:

\[
f(\pi, y) = M_{Pl}^2 f^{(2)}(\pi) + M_{Pl} f^{(1)}(\pi) + f^{(0)}(\pi, y), \tag{11}
\]

\[
K(\pi, \pi^+, y, y^+) = M_{Pl}^2 d^{(2)}(\pi, \pi^+) + M_{Pl} d^{(1)}(\pi, \pi^+) + d^{(0)}(\pi, \pi^+, y, y^+), \tag{12}
\]

where

\[
d^{(0)} = y^a \delta^b y^+_b + \left( \sum_i c^{(0)}_m(\pi, \pi^+) g^{(2)}_m(y) + h.c. \right), \tag{13}
\]

\[
f^{(0)} = \sum_i c^{(0)}_m(\pi) g^{(3)}(y), \tag{14}
\]

and the functions \( g^{(3)}(y) \) and \( g^{(2)}_m(y) \) are trilinear and bilinear polynomials in \( y \) fields, gives rise to the \( \mu \)-terms for the matter superfields in \( d=4 \ N=1 \) potential.

\[
V = \frac{e^G/M_{Pl}^2}{\partial \Phi^A} \left( \frac{\partial^2 G}{\partial \Phi^A \partial \Phi^+_B} \right)^{-1} \frac{\partial G}{\partial \Phi^+_B} - 3 \right) + D - terms, \tag{15}
\]

via the condensate of "hidden sector" fields, namely

\[
\mu_m = m \rho_i \partial \partial_{\pi^+} c^{(0)}_m(\pi, \pi^+), \tag{16}
\]

with

\[
\rho_i = \frac{\partial^2 d^{(2)}(\pi, \pi^+) \partial}{\partial \pi \partial \pi^+} (\ln f^{(2)}(\pi) + d^{(2)}(\pi, \pi^+) + c^{(0)}_m(\pi) g^{(3)}(y)), \tag{17}
\]

\[
m = (\exp(d^{(2)}(\pi, \pi^+)/2)f^{(2)}(\pi)) \tag{18}
\]

being the gravitino mass.

At present we are interested in how the mass terms for the Higgs fields can appear in the effective 4-dimensional theory. Hence, we concentrate on the part of (??) which contains \( H_1 \) and \( H_2 \) superfields. Obviously the \( \mu \)-term is still non-invariant with respect to the anomalous \( U(1)_A \) gauge symmetry (for the Higgs fields we are considering \( p 
eq -q \)). The situation can be improved if we assume the presence of the fields \( \lambda^i \) charged under the anomalous \( U(1)_A \) in the "hidden sector" \( \pi \) and contributing into the lagrangian via the last term of equation (??):

\[
\Gamma(\lambda_i, \lambda^+_i, y, y^+) = \sum_i (\lambda_i)^N H_1 H_2 + h.c. \tag{19}
\]

This leads to the desired result if the fields \( \lambda_i \) have charges equal to \(- (p + q)/N\) under \( U(1)_A \). In this case the parameter \( \mu \) will have the same form given by (??) but with the holomorphic function \( c'(\lambda) = \sum_i (\lambda_i)^N \) in the right-hand side.
3.2 The model

Let us analyze the four dimensional Higgs potential within the given scenario. Using the invariance of $N = 1$ supergravity lagrangian under the Kähler transformations [?] one can formulate the theory in terms of a single function – the superpotential, which contains the term:

$$W' = \frac{\mu}{\sqrt{2}} H_1 H_2,$$

and along with the anomalous and non–anomalous D–terms the following potential for the neutral components of the supersymmetric SM Higgs fields is generated:

$$V = \frac{g}{4}(\xi + q|H_1^0|^2 + p|H_2^0|^2)^2 + \frac{g^\prime}{4}(|H_1^0|^2 - |H_2^0|^2)^2 + \frac{\mu^2}{2}(|H_1^0|^2 + |H_2^0|^2),$$

again we have the definitions: $g \equiv g_A^2$ and $g' \equiv g_W^2 + g_Y^2$. Note that the presence of the term (??) in the Kähler potential does not change the form (??) of the considered D–terms according to (??).

Solutions that correspond to the non–zero VEV of SM Higgs fields are the following:

$$<|H_1^0|^2> = -\frac{\mu^2(2g' + gp - q) + \xi gg'(p + q)}{gg'(p + q)^2}$$

and

$$<|H_2^0|^2> = -\frac{\mu^2(2g' + gq - p) + \xi gg'(p + q)}{gg'(p + q)^2}.$$  

In order to have the physically accepted result the right-hand sides of (??) and (??) must be positive. It is easy to see that it can be realized with the help of $\xi$–term if we suppose that the absolute value of first term in eq. (??) and (??) is less then second one. Thus, the anomalous $U(1)_A$ symmetry again plays a crucial role for the formation of correct values of VEV for SM Higgs fields.

It is easy to see that VEV of $D_A$ - terms, that corresponds to anomalous $U(1)_A$ gauge symmetry in our scenario, is different from zero. In this case extra contribution to the scalar masses arises for fields having nontrivial charges under considered $U(1)_A$ symmetry. From the Eqs. (??), (??) and (??) one obtains:

$$\Delta m_i^2 = Q_i \frac{4\mu^2}{p + q}. \quad (24)$$

This contribution can be useful for the solution of supersymmetric flavor problem, namely the relevant phenomenology of this kind was discussed in [?, ?]. Notice that the same kind of contribution to the scalar masses takes place in the model considered in the previous Section as well.

Hence, we have proven that embedding of the MSSM into the $d = 4 \ N = 1$ superstring models when string scale is of the order of TeV the electroweak symmetry breaking mediated by the Fayet–Illiopoulos term corresponding to an anomalous $U(1)_A$ $D_A$–term can take place.
4 Summary

In this letter we have studied a possible influence of anomalous $U(1)_A$ symmetry on the violation of SM gauge symmetry in the supersymmetric case. Our results show that the value of Fayet–Iliopoulos $\xi$–term is crucial for the generation of correct VEV of Higgs fields when having the low–scale superstring theory. The only important requirement on the value of parameter $\mu$ to have the self–consistent picture of electroweak symmetry breaking is $|\mu| < |\xi|$. In contrast with the other models discussed in the literature the order of parameter $\mu$ is unimportant for the formation of correct vacuum VEVs of the Higgs fields and can be fixed from the low bound of chargino–neutralino section. In both suggested models we have found the extra contribution from VEV of $D_A$–term. This result cane be used for the solution of the supersymmetric flavor problem.

The models considered in this paper have left several unresolved questions both from the theoretical and phenomenological points of view. Namely, it would be interesting to obtain these models using the string theory computation and to examine whether $SU(2)_W \otimes U(1)_Y$ symmetry breaking can take place in the known four dimensional superstring models. The latter procedure can require the inclusion of several $\xi_i$ terms corresponding to anomalous $U(1)^i_A$ symmetries present in type IIB superstring theory.

Acknowledgments We are grateful to Zurab Berezhiani, Durmush Demir, Gia Dvali, Edi Gava, Alexei Gladyshev, Dimitri Gorbunov, Elias Kiritsis, and Goran Senjanovic for interest to this work and helpful discussions. Work of M.T. was supported in part by the Russian Foundation of Fundamental Research, under the grant 99-02-18417.
References


